Benha University Faculty of Engineering- Shoubra Civil Engineering Department Second year Civil Engineering		Final term exam Date: 27/1/2016 Code : MPH251 Mathematics and Numerical analysis Time : 3 hours

Answer all the following questions **No. of questions : two** **Total Mark: 70**

Question 1 **[35 marks]**

(a) Show that $v(x,y) = x + e^{2x}\cos 2y$ is harmonic and find $u(x,y)$ such that $f(z) = u + iv$ is analytic, function express $f\,(z)$ in terms of Z only **[10 marks]**

(b) Evaluate the following integrals: **[10 marks]**

(i) $\oint_c \frac{z^2}{(z^2+4)^2}\,dz$, where **c** is the circle $x^2 + y^2 = 4y$

(ii) $\oint_c \frac{\cos z}{(z-\pi)^2}\,dz$,where **c** is the circle $|z| = 4$

(c) Find a cubic interpolation polynomial which interpolate the function **y = f(x)** at the points (1,9) , (2,26) , (3,55) ,(4,102) .Hence find the value of x which makes **f(x) = 0** by fixed method . **[15 marks]**

Question 2 **[35 marks]**

(a) Fit the function $y = a\sin^2 x + b$ that best fit the data **[10 marks]**
(0, 4.2) , ($\frac{\pi}{6}$, 5.8) , ($\frac{\pi}{4}$, 8.3), ($\frac{\pi}{2}$, 12.5)

(b) Solve the following partial differential equation:

$$\frac{\partial u}{\partial x} = 4\frac{\partial u}{\partial y} \quad , \qquad u(0,y) = 8e^{3y} + 9e^{10y}$$

[10 marks]

(c) Solve $max.\ f = 4y_1 + 6y_2 + 6y_3 + 10$ **[15marks]**

s.t :

$$2y_1 - y_2 + 3y_3 \le 1$$
$$-y_1 + 2y_2 + 2y_3 \le 2$$

$$y_1 \le 0, y_2 \le 0, y_3 \ge 0$$

Good Luck *Dr.Eng. Zaki Ahmed Zaki*

Second year civil Engineering   final term Exam   27/1/2016

Model Answer

(a) Show that $v(x,y)=x+e^{2x}\cos 2y$ is harmonic and find $u(x,y)$ such that $f(z)=u+iv$ is analytic, express $f(z)$ in terms of Z only **[10 marks]**

$v_x = 1+2e^{2x}\cos 2y$; $v_y = -2e^{2x}\sin 2y$; $v_{xx} = 4e^{2x}\cos 2y$; $v_{yy} = -4e^{2x}\cos 2y$

$then\ v_{xx}+v_{yy}=0\ (v\ \ is\ harmonic)$; $u_x = v_y$; $u=\int v_y\,dx$

$u=-e^{2x}\sin 2y + c(y)$; $also$

$u_y=-v_x$; $-2e^{2x}\cos 2y + c'(y) = -1-2e^{2x}\cos 2y$; $c'(y)=-1$

$c(y)=\int -1\,dy = -y+c$; $f(z)=u+iv=(-e^{2x}\sin 2y - y + c)+i(x+e^{2x}\cos 2y)$

$let\ x=z\ and\ y=0\ then\ f(z)=i(z+e^{2z})$

(b) $(i)\ \oint_c \frac{z^2}{(z^2+4)^2}dz$, where **c** is the circle $x^2+y^2=4y$ **[5 marks]**

$\oint_c \frac{z^2}{(z+2i)^2(z-2i)^2}dz$ not defined at z = 2i (inside) and z = -2i (outside)

$$\oint_c \frac{z^2/(z+2i)^2}{(z-2i)^2}dz = 2\pi i(z^2/(z+2i)^2)' = 2\pi i\left(\frac{4iz}{(z+2i)^3}\right) = 2\pi i\left(\frac{1}{8i}\right) = \frac{\pi}{4}$$

(ii) $\oint_c \frac{\cos z}{(z-\pi)^2}dz$,where **c** is the circle $|z|=4$ **[5 marks]**

$$(ii)\int_c \frac{\cos z}{(z-\pi)^2}dz = \frac{2\pi i}{1!}(\cos z)' = \frac{2\pi i}{1!}(-\sin z)|_{z=\pi} = 0$$

(c) Find a cubic interpolation polynomial which interpolate the function y = **f(x)** at the points (1,9) , (2,26) , (3,55) ,(4,102) .Hence find the value of x which makes f**(x) = 0** by fixed method . **[15 marks]**

x	y			
1	9	17	12	
2	26	29	18	6
3	55	47		
4	102			

By Newton forward: y(x)=9+(x-1)17+(x-1)(x-2)12/2 ++(x-1)(x-2)(x-3)6/6

y(x)= $x^3+10x-2$ **,,,by fixed** $x_{n+1}=(2-x_n^3)/10$ **,,,root in (0,1)**

i	x_{n+1}=(root),, x_0=0.5
0	**0.1875**
1	**0.19934**
2	**0.1992078**
3	**0.1992094**

Question 2 **[35 marks]**

(a) Fit the function $y = a\sin^2 x + b$ that best fit the data **[10 marks]**

(0, 4.2) , ($\frac{\pi}{6}$, 5.8) , ($\frac{\pi}{4}$, 8.3), ($\frac{\pi}{2}$, 12.5)

30.8=1.75 a+4 b and 18.1=1. 3125 a +1.75 b then a=8.457 , b=4

x	y	X	Xy	X^2
0	4.2	0	0	0
$\frac{\pi}{6}$	5.8	0.25	1.45	0.0625
$\frac{\pi}{4}$	8.3	0.5	4.15	0.25
$\frac{\pi}{2}$	12.5	1	12.5	1
SUM	**30.8**	**1.75**	**18.1**	**1.3125**

(b) Solve the following partial differential equations: **[10 marks]**

$$\frac{\partial u}{\partial x} = 4\frac{\partial u}{\partial y} \quad ; u(0,y) = 8e^{3y} + 9e^{10y}$$

$$u_x - 4u_y = 0 \quad ; u(0,y) = 8e^{3y} + 9e^{10y}$$

$$let \quad u(x,y) = f(x)g(y) \quad ; f'g = 4f\, g'$$

$$\frac{f'}{4f} = \frac{g'}{g} = \alpha \quad ; \int \frac{f'}{f}dx = \int 4\alpha\, dx \quad , f = e^{4\alpha x + c_1} = Ae^{4\alpha x}$$

$$\int \frac{g'}{g}dy = \int \alpha\, dy \quad , g = Be^{\alpha y}$$

$$u(x,y) = C\, e^{\alpha(4x+y)} + C_1 e^{\alpha_1(4x+y)}$$

$$u(0,y) = 8e^{3y} + 9e^{10y} \quad then \quad C = 8, \alpha = 3, C_1 = 9, \alpha_1 = 10$$

$$the \ \ solution \ is \quad u(x,y) = 8e^{12x+3y} + 9e^{40x+10y}$$

(c) Solve $max.\ f = 4y_1 + 6y_2 + 6y_3 + 10$ **[15marks]**

s.t :

$$2y_1 - y_2 + 3y_3 \le 1$$
$$-y_1 + 2y_2 + 2y_3 \le 2$$
$$y_1 \le 0, y_2 \le 0, y_3 \ge 0$$

Let $y_1 = -y'_1, y_2 = -y'_2$

$$max.\ f = -4y'_1 - 6y'_2 + 6y_3 + 10$$

$$-2y'_1 + y'_2 + 3y_3 + s_1 = 1$$
$$y'_1 - 2y'_2 + 2y_3 + s_2 = 2$$

$$y'_1 \ge 0, y'_2 \ge 0, y_3 \ge 0$$

B.V	**y'₁**	**y'₂**	**y₃**	**S₁**	**S₂**	**Solution**
S_1	**-2**	**1**	**<u>3</u>**	**1**	**0**	**1**
S_2	**1**	**-2**	**2**	**0**	**1**	**2**
f	**4**	**6**	**-6**	**0**	**0**	**10**
y_3	**-2/3**	**1/3**	**1**	**1/3**	**0**	**1/3**
S_2	**7/3**	**-8/3**	**0**	**-2/3**	**1**	**4/3**
f	**0**	**8**	**0**	**2**	**0**	**12**

The optimal solution is: **$(y_1, y_2, y_3) = (0, 0, 1/3)$** **And** **<u>f = 12</u>**

Good Luck *Dr.Eng. Zaki Ahmed Zaki*