Benha University Faculty of Engineering- Shoubra (**Electrical**)Engineering Department		Final term exam Date: **28-1-2016** Mathematics **3A** Code: **EMP 281** Duration : **3 hours**
Answer the following questions	No. of questions : **8**	Total Mark: 80

1- Suppose we randomly select 5 cards without replacement from an ordinary deck of playing cards. What is the probability of getting exactly 2 red cards (i.e., hearts or diamonds)?

2- The probability of getting tails is 60% when tossing a coin 5 times. What is the probability of getting 2 tail and 3 head?

3- Suppose that a number of inquiries arriving at a certain interactive system with arrival rate 12 inquiries per minute. Find the probability of at least 2 inquiries arriving in 3 minute interval.

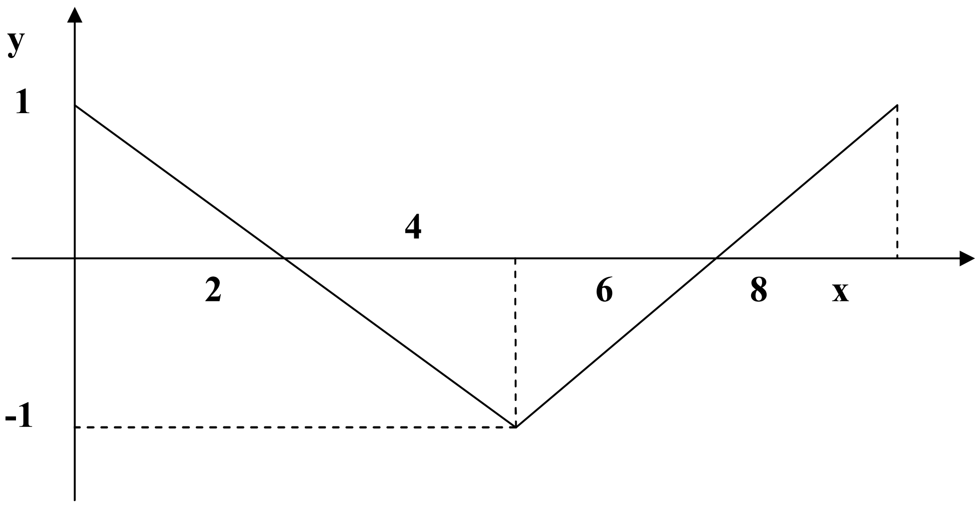
4- If $f(x,y) = cx^2y$ is probability density function, $0 < y < x < 1$, find marginal of x and y. Check for independence, and find $p(y > 1/3 , x > 1/2)$..

5- A weighted die is rolled once and only spots less than 4 facing up and a fair coin is tossed number of times equal to the spot number facing up from rolling the die. Let X denotes the number of head and Y denotes the spots of the die facing up. Discuss the joint distribution

6- For Gamma distribution, find $\mu_r`$ [moment about zero], then deduce μ_3 [3rd moment about mean]

7- Expand in Fourier series $f(x) = |\cos x|, \quad 0 < x < 2\pi$

8-



Expand in Fourier series the illustrated figure with T = 4.

Dr.eng.khaled El Naggar

Model answer

1- This is a hypergeometric experiment in which we know the following:

N = 52; since there are 52 cards in a deck.

k = 26; since there are 26 red cards in a deck.

n = 5; since we randomly select 5 cards from the deck.

x = 2; since 2 of the cards we select are red.

We plug these values into the hypergeometric formula as follows:

$$h(x; N, n, k) = [\,{}^{k}C_{x}\,]\,[\,{}^{N-k}C_{n-x}\,] \,/\, [{}^{N}C_{n}\,]$$

Thus $h(2; 52, 5, 26) = [\,{}^{26}C_{2}\,]\,[\,{}^{26}C_{3}\,] \,/\, [{}^{52}C_{5}\,]$

$$= [\,325\,]\,[\,2600\,] \,/\, [\,2{,}598{,}960\,] = 0.32513$$

Thus, the probability of randomly selecting 2 red cards is 0.32513.

2- Let X: number of tail facing up, therefore $P(X = 2) = {}^{5}C_{2}(0.6)^{2}(0.4)^{3}$

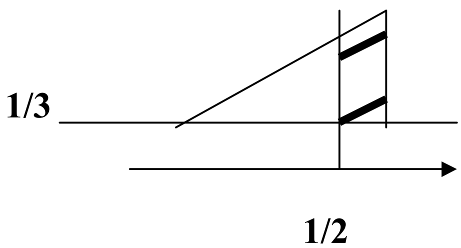
3-X: The number of inquiries arriving at a certain interactive such that $\lambda = 12(3) = 36$,

thus $P(X \geq 2) = 1 - P(X \leq 1) = 1 - \sum_{x=0}^{1} \frac{e^{-\lambda}\lambda^{x}}{x!} = 1 - \sum_{x=0}^{1} \frac{e^{-36}36^{x}}{x!} = 1 - e^{-36} - 36\,e^{-36}$

4) $\int_{0}^{1}[\int_{y}^{1} cx^{2}y\,dx]dy = 1 \Rightarrow c = 10$, $\mathbf{f_x} = \int_{0}^{x} 10x^{2}y\,dy = 5x^{4}$, $\mathbf{f_y} = \int_{y}^{1} 10x^{2}y\,dx = \frac{10}{3}[y - y^{4}]$ **&**

$\mathbf{f(x,y) \neq f_x\, f_y}$

Therefore they are not independent.



p(y > 1/3 , x > 1/2) = $\int_{1/3}^{1/2} [\int_{1/2}^{1} 10x^{2}y\,dx]dy + \int_{1/3}^{1} [\int_{y}^{1} 10x^{2}y\,dx]dy$

5-

x \ y	1	2	3	f_x
0	**1/14**	**1/14**	**1/14**	**3/14**
1	**1/14**	**2/14**	**3/14**	**6/14**
2	**0**	**1/14**	**3/14**	**4/14**
3	**0**	**0**	**1/14**	**1/14**
f_y	**2/14**	**4/14**	**8/14**	**1**

P(y > x) = 10/14

6- The moment generating function can be expressed by

$$E(e^{tx}) = \int_0^\infty e^{tx}\left(\frac{\beta^\alpha}{\Gamma\alpha} x^{\alpha-1}e^{-\beta x}\right)dx = \frac{\beta^\alpha}{\Gamma\alpha}\int_0^\infty x^{\alpha-1}e^{-(\beta-t)x}dx$$

Put $(\beta-t)x = y \Rightarrow dx = \frac{dy}{\beta - t}$, thus

$$E(e^{tx}) = \frac{\beta^\alpha}{(\beta-t)^\alpha \Gamma\alpha}\int_0^\infty y^{\alpha-1}e^{-y}dy = \frac{\beta^\alpha}{(\beta-t)^\alpha}$$

Using the r^{th} derivative, we get $\frac{d^r}{dt^r}\phi(t) = \frac{\beta^\alpha\Gamma(r+\alpha)}{(\beta-t)^{r+\alpha}\Gamma(\alpha)}$

Therefore $\mu_r` = \frac{\Gamma(r+\alpha)}{\beta^r\Gamma(\alpha)}$ and $\mu_3 = E(x-\mu)^3 = \mu_3` - 3\mu_1`\mu_2` + 2[\mu_1`]^3$, where

$$\mu_3` = \frac{\Gamma(3+\alpha)}{\beta^3\Gamma(\alpha)} = \frac{(2+\alpha)(1+\alpha)(\alpha)}{\beta^3},\ \mu_2` = \frac{\Gamma(2+\alpha)}{\beta^2\Gamma(\alpha)} = \frac{(1+\alpha)(\alpha)}{\beta^2},\ \mu_1` = \frac{\Gamma(1+\alpha)}{\beta\Gamma(\alpha)} = \frac{\alpha}{\beta}$$

,

7- This function is even cosine harmonic, therefore $a_0 = \frac{4}{T}\int_0^{T/2} f(x)\,dx = \frac{4}{\pi}\int_0^{\pi/2}\cos(x)\,dx = \frac{4}{\pi}$

$$a_{2n} = \frac{4}{T}\int_0^{T/2} f(x)\cos\left(\frac{2n\pi x}{T}\right)dx = \frac{4}{\pi}\int_0^{\pi/2}\cos(x)\cos(2nx)\,dx = \frac{4\cos(n\pi)}{\pi(2n-1)(2n+1)}\ ,$$

$$b_{2n} = 0$$

Therefore $|\mathbf{cosx}| = \frac{2}{\pi} + \sum_{n=1}^{\infty} \frac{4\cos(n\pi)}{\pi(2n-1)(2n+1)} \cos 2nx$

Put $x = \frac{\pi}{2}$, thus

$-\frac{1}{2} = \sum_{n=1}^{\infty} \frac{\cos(n\pi)}{(2n-1)(2n+1)} \cos(n\pi) \Rightarrow -\frac{1}{2} = \sum_{n=1}^{\infty} \frac{1}{(2n-1)(2n+1)} = \frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} - ....$

By Parseval's theorem, $\frac{2}{T}\int_0^T [f(x)]^2 dx = \frac{a_0^{\ 2}}{2} + \sum_{n=1}^{\infty} a_{2n}^2 = \frac{1}{\pi}\int_0^{\pi} (1 + \cos 2x) dx = 1$,

Therefore $1 = \frac{8}{\pi^2} + \sum_{n=1}^{\infty} \frac{16}{\pi^2 (2n-1)^2 (2n+1)^2} \Rightarrow \sum_{n=1}^{\infty} \frac{1}{(2n-1)^2 (2n+1)^2}$ = [1 - $\frac{8}{\pi^2}$]$\frac{\pi^2}{16}$

- Intended Learning Outcomes of Course (ILOS)

a- Knowledge and Understanding

On completing this course, students will be able to:

a- 1 - Recognize concepts and theories of mathematics and sciences (a1)

a- 2 - Recognize methodologies of solving engineering problems, data collection interpretation. (a6)

b- Intellectual Skills

At the end of this course, the students will be able to:

b- 1 - Select appropriate mathematical and computer-based methods for modeling and analyzing problems. (b1)

b- 2 - Select appropriate solutions for engineering problems based on analytical thinking. (b3)

b- 3 - Solve engineering problems, often on the basis of limited and possibly contradicting information. (b8)

c- Professional Skills

On completing this course, the students are expected to be able to:

c- 1 - Apply knowledge of mathematics, science, information technology, design, busine

c- 2 - Apply numerical modeling methods to engineering problems. (c7)

d- General Skills

At the end of this course, the students will be able to:

d-1- Work in stressful environment and within constraints. (d2)

Questions	Total marks	Achieved ILOS	Questions	Total marks	Achieved ILOS
Q1	**10**	**b1**	**Q5**	**10**	**b3**
Q2	**10**	**a1**	**Q6**	**10**	**c1**

Q3	**10**	**a2, c1**	**Q7**	**10**	**a1, b1**
Q4	**10**	**b2**	**Q8**	**10**	**c1**

Board of examiners: Dr. eng. Khaled El Naggar