Benha University Faculty of Engineering- Shoubra Mechanical Engineering Department Second year Mech. Eng(Power) تخلفات		Final term exam Date:26 /12/ 2015 Code: EMP 231 Mathematics and Computers(A) Duration : 3 hours
Answer all the following questions	No. of questions : **Two**	Total Mark: **70**

Question 1 **[35 marks]**

[**a**] Show that $v = e^{2x}\cos 2y$ is harmonic and find $u(x,y)$ such that $f(z) = u + iv$ is analytic, express $f(z)$ in terms of Z only. **[10 marks]**

[**b**] Find $u(x,y)\, and\, v(x,y)$ for $f(z) = ie^{2z}$ **[5 marks]**

[c] Find zeros of $z^5 + 32 = 0$ **[5 marks]**

[d] Evaluate the following integral: $\int_c \frac{z}{z^2-1}dz$; **[15 marks]**

in the following cases: $c\ is: (i)\ |z-1| = 1\ (ii)\ |z| = \frac{1}{2}\ (iii)\ |z| = 2$

Question 2 **[35 marks]**

[a] Find the Fourier series of the function

$$f(x) = \begin{cases} -2-x & ; -2 \le x \le 0 \\ 2-x & ; 0 \prec x \le 2 \end{cases}$$ And f(x+4) =f(x) **[10marks]**

[b] State the types of linear second order partial differential equations with examples **[10 marks]**

[c] Solve the following partial differential equations: **[15 marks]**

$$(i)\ u_x - 2u_y - u = 0\ ;\ u(x,0) = 4e^{3x}$$

$$(ii)\ u_{xx} + u_{xy} - 6u_{yy} = \sin(2x+y)$$

Good Luck *Dr.Eng. Zaki Ahmed Zaki*

Second year Mech. Engineering (power) final term Exam (26-12- 2015)

Model Answer

[**a**] Show that $v = e^{2x}\cos 2y$ is harmonic and find u such that $f(z) = u + iv$ is analytic, **[10 marks]**

$$v_x = 2e^{2x}\cos 2y \ , v_{xx} = 4e^{2x}\cos 2y \ , v_y = -2e^{2x}\sin 2y \ , v_{yy} = -4e^{2x}\cos 2y$$

$$\because v_{xx} + v_{yy} = 0 \ ; \textit{then } v \textit{ is harmonic}$$

$$\because f(z) \textit{ is analytic then } u_x = v_y \to u = \int v_y\, dx = \int -2e^{2x}\sin 2y\, dx$$

$$u = -e^{2x}\sin 2y + c(y); \ \because u_y = -v_x \to -2e^{2x}\cos 2y + c'(y) = -2e^{2x}\cos 2y$$

$$c'(y) = 0 \to c(y) = constant \ \textit{may be neglected}$$

$$\therefore u = -e^{2x}\sin 2y \to f(z) = u + iv = -e^{2x}\sin 2y + i\, e^{2x}\cos 2y \ ;$$

[**b**] Find $u(x, y)$ and $v(x, y)$ for $f(z) = ie^{2z}$ **[5 marks]**

$$f(z) = ie^{2z} = ie^{2(x+iy)} = ie^{2x}e^{2iy} = ie^{2x}(\cos 2y + i\sin 2y)$$

$$f(z) = -e^{2x}\sin 2y + ie^{2x}\cos 2y \to u = -e^{2x}\sin 2y \quad and \ v = e^{2x}\cos 2y$$

[**c**] Find zeros of $z^3 + 32 = 0$ **[5 marks]**

$$z^5 = -32 = 32(-1) = 32e^{i\pi} \to z = 2e^{i\left(\frac{\pi + 2k\pi}{5}\right)} \ ; k = 0,1,2,3,4$$

[d] Evaluate the following integral: $\int_c \frac{z}{z^2 - 1} dz$; **[15 marks]**

in the following cases: $c \ is : (i)\ |z - 1| = 1 \ (ii)\ |z| = \frac{1}{2}$

$$\int_c \frac{z}{z^2 - 1} dz = \int \frac{z}{(z-1)(z+1)} \ ;$$

$$(i)\ |z - 1| = 1 \ , z = 1 (inside), z = -1 (outside)$$

$$\int \frac{z}{(z-1)(z+1)} dz = \int \frac{\frac{z}{(z+1)}}{(z-1)} dz = 2\pi i \ \frac{1}{(1+1)} = \pi i$$

$$(ii)\ |z| = \frac{1}{2} \ , z = 1, -1 \ are\ outside\ then \int \frac{z}{(z-1)(z+1)} dz = 0$$

Question 2

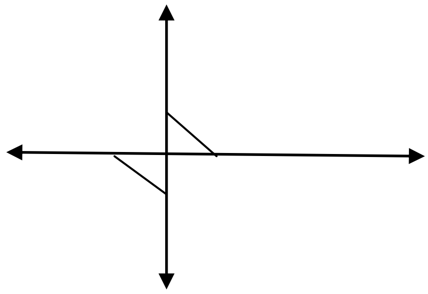
(a) Find the Fourier series of the function **[10marks]**

$$f(x)=\begin{cases}-2-x & ;-2\le x\le 0\\ 2-x & ;0\prec x\le 2\end{cases}$$ And f(x+4) =f(x)

It is odd then $a_0=0;a_n=0$

$$b_n=\frac{2}{2}\int_0^2(2-x)\sin\frac{n\pi}{2}x\,dx\ ;Integrate\ by\ parts$$

$$b_n=\left[\frac{-2}{n\pi}(2-x)\cos\frac{n\pi}{2}x-\frac{4}{n^2\pi^2}\sin\frac{n\pi}{2}x\right]_0^2=\frac{4}{n\pi}$$



the fourier series is

$$f(x)=\sum_{n=1}^{n=\infty}b_n\ \sin\frac{n\pi}{2}x=\sum_{n=1}^{n=\infty}\frac{4}{n\pi}\ \sin\frac{n\pi}{2}x$$

(b) State the types of linear second order partial differential equations with examples **[10marks]**

$u_t=cu_{xx}(one\ \dim.heat\ equation)$; $u_t=c(u_{xx}+u_{yy})two\ \dim.heat\ equation$ $(parabolic\ equation)$

$u_{tt}=c^2u_{xx}(wave\ equation)$ $(hyperbolic\ equation)$

$u_{yy}+u_{xx}=0(laplace\ equation)$ $(elliptic\ equation)$

(c) Solve the following partial differential equations: [15marks]

$(i)\ u_x-2u_y-u=0\ ;u(x,0)=4e^{3x}$

$let\ u(x,y)=pe^{ax+by}\ ;u(x,0)=pe^{ax}=4e^{3x}\ ;p=4\ and\ a=3$

$sub.in\ the\ eq.(a-2b-1)pe^{ax+by}=0\ then\ a-2b-1=0\ ,b=1$

$the\ solution\ is\ u(x,y)=4e^{3x+y}$

$(ii)u_{xx}-4u_{xy}+4u_{yy}+u_x-u_y-2u=0\ (classify)$

$A=1;B=-4,C=4\ then\ B^2-4AC=16-16=0(Parabolic)$

$k^2-4k+4=0\ ,k=2,2\ then\ u(x,y)=f_1(y+2x)+xf_2(y+2x)$

$let\ v=y+2x\ ,v_x=2\ ;v_y=1\ and\ u_x=\frac{du}{dv}v_x\ ,u_y=\frac{du}{dv}v_y$

$sub.\ in\ the\ eq.\ \frac{du}{dv}v_x-\frac{du}{dv}v_y-2u=0\ ,\frac{du}{dv}-2u=0\ then\ u=c_1e^{2v}=c_1e^{2(y+2x)}$

$the\ solution\ is\ u(x,y)=c_1e^{2(y+2x)}$