Benha University
Faculty of Engineering- Shoubra
(**Electrical**)Engineering Department

Final term exam Date: **18-1-2016**
Mathematics **3A** Code: **EMP 271**
Duration : **3 hours**

Answer the following questions No. of questions : **8** Total Mark: 80

1- A batch of 100 printed circuit cards is populated with semiconductor chips.20 of these are selected without replacement for function testing. If the original batch contains 30 defective cards. What is the probability of exactly 5 cards are defective from the sample.

2- An exam consists of 20 multiple choice questions with 5 possible responses to each question. Find the probability of answering at least 4 questions correctly.

3-There are 3 calls coming per minute into a hotel reservation center. Find the probability that at least 1 call come in a given 1 hour.

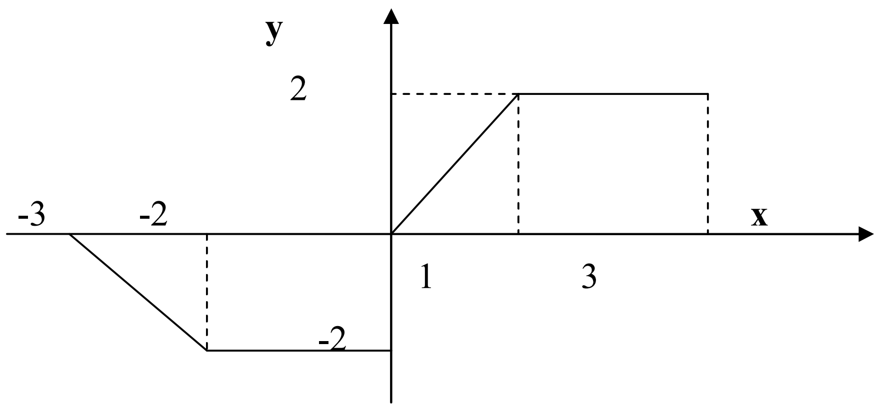
4- If $f(x,y) = cx^2 + \frac{xy}{3}$ is probability density function, $0 < x < 1$, $0 < y < 2$, find marginal of x & y. Check for independence, and find $p(x + y > 1)$.

5-Suppose that 2 batteries are randomly chosen without replacement from the following group of 12 batteries 3 new – 4 used (working) – 5 defective. Let X denote the number of new batteries chosen and Y denote the number of used batteries chosen. Discuss the joint distribution.

6- For exponential distribution, find μ_r` [moment about zero], then deduce μ_3 [3rd moment about mean].

7- Expand in Fourier series $f(x) = |\sin x|$, $0 < x < 2\pi$, deduce $\frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} - .....$

8-



Expand in Fourier series the illustrated figure with T = 3.

Dr. eng. Khaled El Naggar

Model answer

1- This is a hypergeometric experiment in which we know the following:

X : number of defective cards, therefore N = 100; k = 30; n = 20; x = 5

We plug these values into the hypergeometric formula as follows:

$$h(x; N, n, k) = [\,{}^{k}C_{x}\,]\,[\,{}^{N-k}C_{n-x}\,] / [\,{}^{N}C_{n}\,]$$

Thus $h(5; 100, 20, 30) = [\,{}^{30}C_{5}\,]\,[\,{}^{70}C_{15}\,] / [\,{}^{100}C_{20}\,]$

2- $P(x \geq 4) = 1 - P(x \leq 3) = 1 - \sum_{x=0}^{3} {}^{20}c_{x}(0.2)^{x}(0.8)^{20-x}$

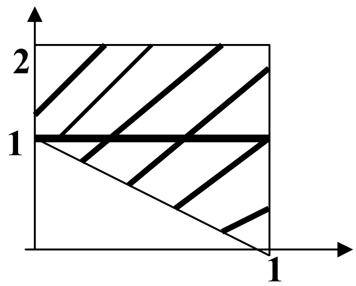
3- Let X is the number of calls coming into hotel reservation and $\lambda = 180$ / hour therefore

$P(x \geq 1) = 1 - P(x=0) = 1 - \frac{\lambda^{0}e^{-\lambda}}{0!} = 1 - \frac{1}{e^{180}}$

4- $\int_0^2 [\int_0^1 cx^2 + \frac{xy}{3} dx]dy = 1 \Rightarrow c = 24$, $f_x = \int_0^2 (24x^2 + \frac{xy}{3})dy = 12x(1-x)^2$,

$f_y = \int_0^1 (24x^2 + \frac{xy}{3})dy = 12y(1-y)^2$, & $f(x,y) \neq f_x f_y$

Therefore they are not independent.



$$p(x+y>1) = \int_0^1 [\int_{1-y}^{1} (24x^2 + \frac{xy}{3})dx]dy + \int_1^2 [\int_0^1 (24x^2 + \frac{xy}{3})dx]dy = \frac{65}{72}$$

5-

y \ x	0	1	2	f_y
0	$\frac{20}{132}$	$\frac{30}{132}$	$\frac{6}{132}$	$\frac{56}{132}$

1	$\frac{40}{132}$	$\frac{24}{132}$	**0**	$\frac{64}{132}$
2	$\frac{12}{132}$	**0**	**0**	$\frac{12}{132}$
f_x	$\frac{72}{132}$	$\frac{54}{132}$	$\frac{6}{132}$	**1**

6- $\mu_r` = \frac{r!}{\lambda^r}$, therefore $\mu_3 = E(x-\mu)^3 = \mu_3` - 3\mu_1`\mu_2` + 2[\mu_1`]^3$

7- This function is even cosine harmonic, therefore $a_0 = \frac{4}{T}\int_0^{T/2} f(x)\,dx = \frac{4}{\pi}\int_0^{\pi/2} \sin(x)\,dx = \frac{4}{\pi}$

$a_{2n} = \frac{4}{T}\int_0^{T/2} f(x)\cos(\frac{2n\pi x}{T})\,dx = \frac{4}{\pi}\int_0^{\pi/2} \sin(x)\cos(2nx)\,dx = -\frac{2}{\pi(2n-1)(2n+1)}$, $b_{2n} = 0$

Thus $|\sin x| = \frac{a_0}{2}\sum_{n=1}^{\infty} a_{2n}\cos(2nx) = \frac{2}{\pi} - \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{1}{(2n-1)(2n+1)}\cos(2nx)$, put x = 0, therefore

$\sum_{n=1}^{\infty}\frac{1}{(2n-1)(2n+1)} = 1$

Intended Learning Outcomes of Course (ILOS)

a- Knowledge and Understanding

On completing this course, students will be able to:

a.1) Recognize concepts and theories of mathematics and sciences, appropriate to the discipline. (a.1)

a.2) Recognize methodologies of solving engineering problems. (a.5)

b- Intellectual Skills

At the end of this course, the students will be able to:

b.1) Select appropriate mathematical and computer-based methods for modeling and analyzing problems. (b.1)

b.2) Select appropriate solutions for engineering problems based on analytical thinking. (b.2)

b.3) Solve engineering problems, often on the basis of limited and possibly contradicting information. (b.7)

c- Professional Skills

On completing this course, the students are expected to be able to:

c.1) Apply knowledge of mathematics, science, information technology, design, business context and engineering practice to solve engineering problems. (c.1)

c.2) Apply numerical modeling methods to engineering problems. (c.7)

Question	Marks	Achieved ILOS
1	10	a1,b1
2	10	a1,c1
3	10	a5, b2,b7
4	10	c7,b7
5	10	a1,c7
6	10	a5,b1
7	10	c1,b2
8	10	a1,a5