Benha University
Faculty of Engineering at Shoubra
Electrical Engineering Department
Third Year Communications

Final Term Exam
Date: Monday 23/1/2017
Subject: Electromagnetic Waves
Model Answer

- **Answer all the following questions**
- **Illustrate your answers with sketches when necessary**
- **No. of questions: 4**
- **Total Mark: 90 Marks**

Question 1	**25 marks**

a- A 50 V voltage generator at 20 MHz is connected to the plates of an air dielectric parallel-plate capacitor with plate area 2.8 cm^2 and separation distance 0.2 mm. Find the maximum value of (i) displacement current density and (ii) displacement current.

Solution (10 Marks)

(i) $J_{ds} = j\omega D_s \rightarrow |J_{ds}|_{max} = \omega\varepsilon E = \omega\varepsilon \frac{V_s}{d}$

$$= \frac{10^{-9}}{36\pi} \times \frac{2\pi \times 20 \times 10^6 \times 50}{0.2 \times 10^{-3}}$$

$$= 277.8 \text{ A/m}^2$$

(ii) $I_{ds} = J_{ds} \cdot S = \frac{1000}{3.6} \times 2.8 \times 10^{-4} = 77.78 \text{ Ma}$

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b- Let the fields, E(z, t) = 1800 cos($10^7\pi$t - βz)a_x V/m and H(z, t) = 3.8 cos($10^7\pi$t - βz)a_y A/m, represent a uniform plane wave propagating at a velocity of 1.4×10^8 m/s in a perfect dielectric. Find: (i) β. (ii) λ. (iii) η. (iv) μ_r. (v) ε_r.

Solution (15 Marks)

(i) $\beta = \frac{\omega}{v} = \frac{10^7\pi}{1.4\times10^8} = 0.224 \text{ m}^{-1}$

(ii) $\lambda = \frac{2\pi}{\beta} = 28 \text{ m}$

(iii) $\eta = \frac{|E|}{|H|} = 474\ \Omega$

(iv) We have two equations in two unknowns, μ_r and ε_r: $\eta = \sqrt{\frac{\mu_0\mu_r}{\varepsilon_0\varepsilon_r}}$ and $\beta = \omega\sqrt{\mu_0\mu_r\varepsilon_0\varepsilon_r}$

$$\varepsilon_r = \frac{\beta^2}{\omega^2\mu_0\mu_r\varepsilon_0} \Rightarrow \eta = \sqrt{\frac{\mu_0\mu_r}{\varepsilon_0\frac{\beta^2}{\omega^2\mu_0\mu_r\varepsilon_0}}} = \frac{\mu_0\mu_r\omega}{\beta} \Rightarrow \mu_r = \frac{\eta\beta}{\omega\mu_0} = 2.69$$

(v) $\varepsilon_r = \frac{\beta^2}{\omega^2\mu_0\mu_r\varepsilon_0} = 1.7$

Question 2	**20 marks**

a- A linearly polarized uniform plane wave is propagating in a homogenous medium. The following information is provided:

- The operating frequency is 10 GHz.
- The attenuation constant in the medium is $4\pi\times10^2$ Nepers/mm.
- $|E|/|H| = 0.0889\ \Omega$.
- The magnetic field phasor lags the electric field one by 45°.

(i) What is the wavelength in the medium?

(ii) What distance must the wave propagate for its time-average power density to be reduced by 50%?

(iii) Calculate the magnetic permeability and the conductivity of the medium?

Solution (15 Marks)

(i) The electric and magnetic field are 45° out of phase; hence , the medium is a good conductor.

Thus, it is $\beta = \alpha = 4\pi \times 10^5\ \mathrm{m}^{-1}$ ➔ $\lambda = \frac{2\pi}{\beta} = 5 \times 10^{-6}$ m

(ii) $| < S(t,z) > | = \frac{1}{2|\eta|}|E|^2 e^{-2\alpha z} \cos\theta_\eta$

$| < S(t,z+d) > | = 0.5 \times | < S(t,z) > |$

$\therefore e^{-2\alpha z} = 0.5$ ➔ $d = \frac{-\ln 0.5}{2\times4\pi\times10^5} = 0.2758\ \mu\mathrm{m}$

(iii) $|\eta| = \sqrt{\frac{\omega\mu}{\sigma}} = 0.0889$ and $\alpha = \sqrt{\frac{\omega\mu\sigma}{2}} = 4\pi \times 10^5$

$$\frac{\alpha}{|\eta|} = \frac{\sigma}{\sqrt{2}} \rightarrow \sigma = \sqrt{2}\frac{\alpha}{|\eta|} = 2 \times 10^7\ \mathrm{S/m}$$

$$\alpha|\eta| = \frac{\omega\mu}{\sqrt{2}} \rightarrow \mu = \sqrt{2}\frac{\alpha|\eta|}{\omega} = 25.1 \times 10^{-7}\ \mathrm{H/m}$$

b- It is required a minimum field of 0.25 mV/m for AM station covering the area of a city. What is the power density ($\mathrm{W/m^2}$) associated with this minimum field?

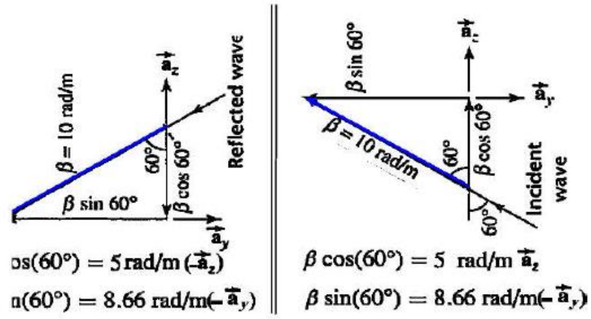
Solution (5 Marks)

Power density = $| < S > | = \frac{1}{2|\eta|}|E|^2$

$E = 25\ \mathrm{mV/m},\ |\eta| = 377\ \Omega$

$| < S > | = 0.829 \times 10^{-7}\ \mathrm{W/m^2}$

Q3 solution



$$\beta = \frac{\omega}{c} = \frac{3 \times 10^9}{3 \times 10^8} = 10 \text{ rad/m}$$

$$\tilde{\vec{\mathbf{E}}}_i = 37.7e^{-j\beta z'}\,\vec{\mathbf{a}}_x = 37.7e^{-j5z}e^{j8.66y}\,\vec{\mathbf{a}}_x \text{ V/m}$$

Using Maxwell's equation, $\nabla \times \tilde{\vec{\mathbf{E}}} = -j\omega\mu_0\tilde{\vec{\mathbf{H}}}$

$$\tilde{\vec{\mathbf{H}}}_i = [0.05\,\vec{\mathbf{a}}_y + 0.087\,\vec{\mathbf{a}}_z]e^{-j5z}e^{j8.66y} \text{ A/m}$$

$$\tilde{\vec{\mathbf{E}}}_r = -37.7e^{-j\beta z''}\,\vec{\mathbf{a}}_x = -37.7e^{j5z}e^{j8.66y}\,\vec{\mathbf{a}}_x \text{ V/m}$$

$$\tilde{\vec{\mathbf{H}}}_r = [0.05\,\vec{\mathbf{a}}_y - 0.087\,\vec{\mathbf{a}}_z]e^{j5z}e^{j8.66y} \text{ A/m}$$

Summing the incident and the reflected $\mathbf{E}$ fields in free space the total $\vec{\mathbf{E}}$ field

$$\tilde{\vec{\mathbf{E}}} = -37.7(e^{j5z} - e^{-j5z})e^{j8.66y}\,\vec{\mathbf{a}}_x$$

$$= -j75.4\sin(5z)e^{j8.66y}\,\vec{\mathbf{a}}_x$$

Similarly, we obtain the total $\tilde{\vec{\mathbf{H}}}$ field as

$$\tilde{\vec{\mathbf{H}}} = 0.1\cos(5z)e^{j8.66y}\,\vec{\mathbf{a}}_y - j0.174\,\sin(5z)e^{j8.66y}\,\vec{\mathbf{a}}_z$$

We can express these fields in the time domain as

$$E_x(x, y, z, t) = 75.4\,\sin(5z)\,\sin(3 \times 10^9 t + 8.66y) \text{ V/m}$$

$$H_y(x, y, z, t) = 0.1\cos(5z)\,\cos(3 \times 10^9 t + 8.66y) \text{ A/m}$$

$$H_z(x, y, z, t) = 0.174\sin(5z)\sin(3 \times 10^9 t + 8.66y) \text{ A/m}$$

The phase velocity can be obtained by setting

$$3 \times 10^9 t + 8.66y = \text{constant}$$

differentiating with respect to t

$$u_{py} = \frac{dy}{dt} = -3.46 \times 10^8 \text{ m/s}$$

the wave propagates in the negative y direction.

the group velocity $u_{py}u_{gy} = u_p^2$

$$u_{gy} = -\frac{[3 \times 10^8]^2}{3.46 \times 10^8} = -2.6 \times 10^8 \text{ m/s}$$

in the negative y direction.

Q 4 – Solution

The field components for TM_m mode are:

$$H_y = H_0 \cos\left(\frac{m\pi}{b}x\right) e^{-j\beta_z z},$$

$$E_x = \frac{\beta_z}{\omega\epsilon} H_0 \cos\left(\frac{m\pi x}{b}\right) e^{-j\beta_z z},$$

$$E_z = -\frac{m\pi}{j\omega\epsilon b} H_0 \sin\left(\frac{m\pi x}{b}\right) e^{-j\beta_z z},$$

$$\beta_z = \left[\omega^2\mu\epsilon - \left(\frac{m\pi}{b}\right)^2\right]^{\frac{1}{2}},$$

Hence the tangential electric field E_z is proportional to:

$$\sin\left(\frac{m\pi x}{b}\right)$$

Where m=2 for the TM_2 mode. and b=0.07m=7cm.

The peaks of E_z occur when:

$$\sin\left(\frac{m\pi x}{b}\right) = \pm 1 \quad \text{(with } x \text{ positive)}$$

i.e. when:

$$\frac{m\pi x}{b} = (\pi/2),\ (3\pi/2) \quad \text{(with m=2 and b=7cm)}$$

∴ The first peak occurs at:

x = (b/4) = 1.75 cm.

and the second peak occurs at:

x = (3b/4) = 5.25 cm.