Benha University
Faculty of Engineering- Shoubra
Engineering Mathematics & physics dep.
Final term Examination

Subject: Mathematics 1A
Code: EMP 011
Date: 5/1/2019
Duration : 3 hours

Question 3 **[25]**

a) Find the first derivative $\frac{dy}{dx}$ for the following functions **[12]**

i- $y = 3\sin^3 t,\ x = 2\cos^3 t$, ii- $e^x \sin(y) = e^y \cos(x)$,

iii- $f(x) = e^{5x^2} (\text{arcsec}(x))^8 (7 - \text{cosech}\, 3x)^{-12}$

b) Find $y^{(18)}$ for the function $y = \frac{3-5x}{6x+8}$ **[5]**

c) Evaluate the following integrals $\int [8^{3x} + x^2 e^{-5x^3} - \cot(4x) + \frac{x^3}{\sqrt{x-5}}]dx$ **[8]**

Question 4 **[25]**

a) If the hypotenuse of the right triangle is given, show that the area is maximum when the triangle is isosceles. **[5]**

b) Find equation of tangent and normal for the curve $y = \frac{[\ell n\, x]^x}{2^{\ell n(x)-1}}$ at $x = e$ **[8]**

c) Evaluate the following limits **[12]**

i) $\lim_{x\to\infty} \frac{5^x + 2^x}{9^x}$, ii) $\lim_{x\to\infty} x^{1/\sqrt{x}}$, iii) $\lim_{x\to 0} [\frac{1}{x} - \frac{1}{e^x - 1}]$, iv) $\lim_{x\to\infty} e^{-x}\, \ell n\, x$

مع تمنياتنا بالتوفيق   د.م. خالد النجار

Model answer

Answer of Question 4

a) Since the hypotenuse of the right triangle is constant, therefore $x^2 + y^2 = c^2 \Rightarrow$

$2x + 2yy` = 0 \Rightarrow y` = -x/y$

Area: $A = \frac{1}{2}xy \Rightarrow \frac{dA}{dx} = \frac{1}{2}(xy`+y) = \frac{1}{2}(x(\frac{-x}{y}) + y) = 0 \Rightarrow x^2 = y^2 \Rightarrow x = y$

Therefore the area is maximum when the triangle is isosceles.

b) $y` = \dfrac{[\ell n\, x]^x [\ell n(\ell n x) + x(\frac{1/x}{\ln x})][2^{\ell n(x)-1}] - [\ell n\, x]^x [\frac{1}{x}][2^{\ell n(x)-1}]\, \ell n 2}{[2^{\ell n(x)-1}]^2}$

at x = e, slope of the tangent = $y`(1) = 1 - \frac{\ell n 2}{e} = \frac{e - \ell n 2}{e}$

Therfore the point of contact is (e, 1) and hence the equation of tangent is $\frac{y-1}{x-e} = \frac{e-\ell n 2}{e}$

Also equation of normal is $\frac{y-1}{x-e} = \frac{e}{\ell n 2 - e}$

c-i) $\lim\limits_{x\to\infty} \frac{5^x + 2^x}{9^x} = \lim\limits_{x\to\infty} \frac{5^x(1+(\frac{2}{5})^x)}{9^x} = \lim\limits_{x\to\infty} (\frac{5}{9})^x = 0$

ii) This limit of the indeterminate form ∞^0. Let $y = x^{1/\sqrt{x}} \Rightarrow \ell n\, y = \frac{\ell n\, x}{\sqrt{x}}$ Hence L'Hopital rule is ready to be used such that $\lim\limits_{x\to\infty} \ell n\, y$ =

$\lim\limits_{x\to\infty} \frac{\ell n\, x}{\sqrt{x}} = \frac{\infty}{\infty} = \lim\limits_{x\to\infty} \frac{1/x}{1/(2\sqrt{x})} = \lim\limits_{x\to\infty} \frac{2}{\sqrt{x}} = 0$.

Therefore $\lim\limits_{x\to\infty} x^{1/\sqrt{x}} = \lim\limits_{x\to\infty} e^{\ell n\, y} = e^{\lim\limits_{x\to\infty} \ell n\, y} = e^0 = 1$

iii) $\lim\limits_{x\to 0}\left[\dfrac{1}{x}-\dfrac{1}{e^{x}-1}\right]=\lim\limits_{x\to 0}\left[\dfrac{e^{x}-1-x}{x(e^{x}-1)}\right]=\lim\limits_{x\to 0}\left[\dfrac{e^{x}-1}{xe^{x}+(e^{x}-1)}\right]=\lim\limits_{x\to 0}\left[\dfrac{e^{x}}{xe^{x}+2e^{x}}\right]=\dfrac{1}{2}$

iv) $\lim\limits_{x\to\infty} e^{-x}\,\ell n\, x=\lim\limits_{x\to\infty}\dfrac{\ell n\, x}{e^{x}}=\lim\limits_{x\to\infty}\dfrac{1}{xe^{x}}=0$