Benha University
Faculty of Engineering- Shoubra
E.S.E. Program
Final Examination

Subject: Mathematics 1
Code: EMP 101
Date: 5th Jan. 2019
Duration : 2 hours

- **Answer all the following questions**
- **Illustrate your answers with sketches when necessary.**
- **The Exam. Consists of one Page**
- **No. of Questions: 2**
- **Total Mark:** 40 Marks

Question 1 [25]

1- Find the first derivative for the function $y = \cot(\sec(x^2)) + \csc(e^{4x})\tan(x) + e^{x^2}$ [9]

2- Evaluate the following limits a) $\lim_{x\to 0}\frac{1-\cos x}{x^2}$, b) $\lim_{x\to 1}\left[\frac{1}{\ell n\, x} - \frac{1}{x-1}\right]$ [10]

3- Evaluate the n^{th} derivative of a) $f(x) = \cos(5x)\ \sin(2x)$, b) $g(x) = 5^{3x}$ [6]

Question 2 [15]

Answer 3 only of the following

1- Using the binomial theorem, expand $(2-3x)^{1/2}$ [5]

2- Resolve $\frac{x-1}{x^2+2x+1}$ into partial fraction [5]

3- Prove $1^2 + 2^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$ using M.I. [5]

4- Solve $2x + 3y = 7$, $x + 2y = 5$ using Gauss Elimination [5]

Dr. eng. Khaled El Naggar

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Answer of Q1

1- $y = \cot(\sec(x^2)) + \csc(e^{4x})\tan(x)$, therefore

$y` = -[\csc^2(\sec(x^2))][\sec(x^2)\tan(x^2)][2x] - [\csc(e^{4x})\cot(e^{4x})][4e^{4x}]\tan x + \csc(e^{4x})\sec^2(x) + 2x e^{x^2}$

2-a) $\lim_{x\to 0}\frac{1-\cos x}{x^2} = \frac{0}{0} = \lim_{x\to 0}\frac{\sin x}{2x} = \frac{0}{0} = \lim_{x\to 0}\frac{\cos x}{2} = \frac{1}{2}$,

b) This limit of the indeterminate form $(\infty-\infty)$ and we have to rewrite by taking the way of common denominator such that $\lim_{x\to 1}[\frac{1}{\ell n\, x} - \frac{1}{x-1}] = \lim_{x\to 1}[\frac{x-1-\ell n\, x}{(x-1)\,\ell n\, x}] = \frac{0}{0}$, then L'Hospital's rule can be used to evaluate this limit as follows:

$$\lim_{x\to 1}[\frac{x-1-\ell n\, x}{(x-1)\,\ell n\, x}] = \lim_{x\to 1}[\frac{1-1/x}{\ell n\, x + (x-1)/x}] = \lim_{x\to 1}[\frac{x-1}{x\,\ell n\, x + x - 1}]$$

$$= \lim_{x\to 1}[\frac{1}{\ell n\, x + 2}] = \frac{1}{2}$$

3-a) $f(x) = \frac{1}{2}[-\sin(3x) + \sin(7x)]$, therefore

$$f^{(n)}(x) = \frac{1}{2}[-3^n \sin(3x + \frac{n\pi}{2}) + 7^n \sin(7x + \frac{n\pi}{2})],$$

b) $g(x) = 5^{3x}$, therefore $g^{(n)}(x) = 3^n\, 5^{3x}\, [\ell n(5)]^n$

Answer of Q2

1- In using the binomial formula, we let a = 2, b = -3x and n = ½. Substituting in the binomial formula (1), we get:

$$(2-3x)^{1/2} = (2)^{1/2} + (\frac{1}{2})(2)^{-1/2}(-3x) + \frac{(1/2)(-1/2)}{2!}(2)^{-3/2}(-3x)^2 + \frac{(1/2)(-1/2)(-3/2)}{3!}(2)^{-5/2}(-3x)^3 + ...$$

2- $\frac{x-1}{x^2+2x+1} = \frac{x-1}{(x+1)^2} = \frac{A}{(x+1)} + \frac{B}{(x+1)^2} = \frac{A(x+1)+B}{(x+1)^2}$

$A(x+1)+B = x - 1$, put x = -1, therefore B = -2, put x = 0, thus

A + B = -1, from which A = 1, thus $\frac{x-1}{x^2+2x+1} = \frac{1}{(x+1)} - \frac{2}{(x+1)^2}$

3- At n =1, R.H.S. = L.H.S. = 1

At n = k, assume R.H.S. = L.H.S. such that $1^2 + 2^2 + ... + k^2 = \frac{k(k+1)(2k+1)}{6}$,

n = k + 1, L.H.S. = $1^2 + 2^2 + ...+ k^2 + (k+1)^2 = \frac{k(k+1)(2k+1)}{6} + (k+1)^2 =$

$$\frac{k(k+1)(2k+1)+6(k+1)^2}{6} = \frac{(k+1)[k(2k+1)+6(k+1)]}{6}$$

$$= \frac{(k+1)[2k^2+7k+6]}{6} = \frac{(k+1)(k+2)(2k+3)}{6} = \text{R.H.S.}$$

Therefore the formula is true.

4- $\begin{pmatrix} 2 & 3 & 7 \\ 1 & 2 & 5 \end{pmatrix} \square \begin{pmatrix} 2 & 3 & 7 \\ 0 & -1 & -3 \end{pmatrix}$

Therefore y = 3 & 2x + 3y = 7, hence x = -1