Benha University
Faculty of Engineering (Shoubra)
Engineering Mathematics and Physics
Department - Preparatory Year

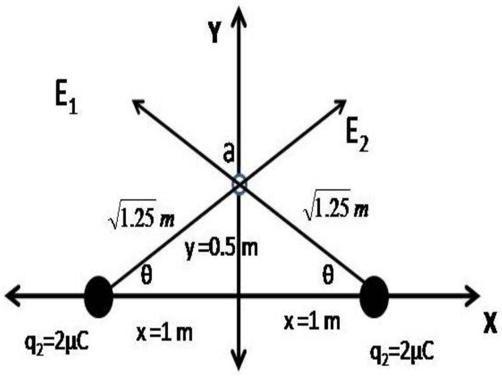
Final Term Exam
Physics
Date: 29 / 12 / 2018
Duration : 3 hours

- Answer all the following question
- Illustrate your answers with sketches when necessary.
- The exam consists of two pages
- No. of questions: 6
- Total Mark: 115 Marks
- The first Page

Question (1) (26 marks)

a) Two 2μC point charges are located on the x axis. One is at x =1 m, and the other is at x = - 1 m. (a) Determine the electric field on the y axis at y = 0.5 m. (b) Calculate the electric force on a - 3μC charge placed on the y axis at y = 0.5 m.

Solution



point a subjects to two electric fields from the two charges 1 and 2 takes the directions as shown in figures the magnitudes of E is

$$E_1 = K\frac{q_1}{r^2} = 9\times10^9 \frac{2\times10^{-6}C}{1.25m^2} = 14.4\times10^3\, N/C = 14.4\,KN/C$$

$$E_2 = K\frac{q_2}{r^2} = 9\times10^9 \frac{2\times10^{-6}C}{1.25m^2} = 14.4\times10^3\, N/C = 14.4\,KN/C$$

$$\mathbf{E_1 = E_2}$$

Since the angle between both E_1 and E_2 with X-axis are the same value of θ the X-components of the two fields are canceled each others

$$E_X = -E_1 \cos\theta + E_2 \cos\theta = 0$$

$$E_Y = E_1 \sin\theta + E_2 \sin\theta = 2\times14.4\,KN/C \times\left(\frac{0.5}{\sqrt{1.25}}\right) = 12.879\,KN/C$$

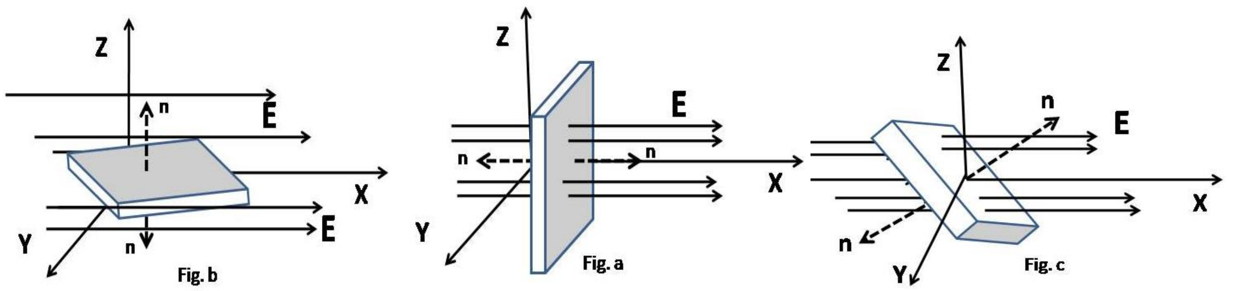
(b) the force acting on - 3μC placed at point a has magnitude

$$F = qE_Y = 3\times10-6C \times 12.879\,KN/C = 38.637\times10-3N = 38.637\,mN$$

This force directed along positive Y-axis

b) An electric field with a magnitude of 3.5 kN/C is applied along the x axis. Calculate the electric flux through a rectangular plane 0.35 m wide and 0.7 m long assuming that (a) the plane is parallel to the yz plane; (b) the plane is parallel to the xy plane; (c) the plane contains the y axis, and its normal makes an angle of 40° with the x axis.

Solution



Electric flux φ is determined by knowing the electric field E and the perpendicular area A_n where

$$\phi = \sum E.A_n = \sum E\,A\cos\theta$$

Where θ is the angle between direction of E and direction of unite vector n perpendicular to A

(a) In fig. a $\phi = EA\cos(0) + EA\cos(180) = EA - EA = 0$

(b) In fig. b $\phi = EA\cos(90) + EA\cos(90) = 0$

(c) In fig. c

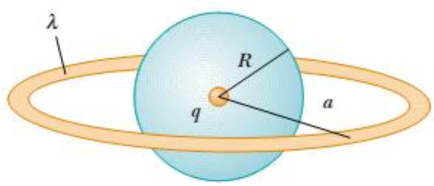
$$\phi = EA\cos(40) + EA\cos(220)$$
$$= EA\cos(40) - EA\cos(40) = 0$$

c) Using principal of charge distributions to find electric field at point on the axis of charged ring and discuss the value of field at points on center of the ring and points of infinity

Solution

The derivation state in lecturer note book

d) A point charge q is located at the center of a uniform ring having linear charge density 3 and radius a, as shown in Figure. Determine the total electric flux through a sphere centered at the point charge and having radius R, where R< a.



Solution

∵ Gauss law state that the total electric flux φ through a closed surface containing an electric charge q is equal to q/ε_o

$$\varphi = \oiint_s E.dA_{\perp} = \frac{q_{inside}}{\varepsilon_\circ}$$

The total electric flux through a sphere of radius R is the sum of the electric flux due to point charge q say φ_1 and duet the surrounded charged ring say φ_2 where

$$\varphi = \varphi_1 + \varphi_2$$

The flux dueto point charge q is $\varphi_1 = \frac{q}{\varepsilon_\circ}$

where all Gauss conditions are satisfied for closed surface and charge q inside the surface

the flux due to charged ring on through the sphere surface is zero because the charge of the ring is completely outside the sphere surface and condition of Gauss law is not satisfied where the total flux which flow in the surface is equal to that flow out the surface

$$\varphi_2 = 0$$

The total flux is

$$\varphi = \varphi_1 + \varphi_2 = \frac{q}{\varepsilon_\circ} + 0 = \frac{q}{\varepsilon_\circ}$$

<u>Question (2) (16 marks)</u>

a) Define with equations and units: i) Electron-volt. ii) Specific resistance.

i) The Electron Volt is defined as : "The change in potential energy (or the work done) when an electron would move between two points that differ in potential by one volt."

$$\mathbf{W_{a\to b} = q_o (V_a - V_b)}$$
$$\mathbf{1\ eV = 1.6 \times 10^{-19}\ J}$$

ii) Specific resistance is defined as: "the ratio of electric field across the conductor to the current density in the conductor"

$$\rho = \frac{E}{J}$$

$$\rho = R\frac{A}{L} \quad \Omega\,m$$

b) The electric field at a point 6 cm from the surface of a charged metal ball is $2\text{x}10^4$ N/C and the potential at the same point is found to be 1800 V. i) What is the charge of the ball?

$E = 2\text{x}10^4$ N/C

$V = 1800$ V

$r = R + 0.06$ m

$E = k\, q/ r^2$

$V = k\, q/ r$

$V^2 = k^2 q^2/ r^2$

So, $V^2/E = k\, q$

$(1800)^2/ 2\text{x}10^4 = 9\text{x}10^9\ q$

$q = 1.8\ \text{x}10^{-8}$ C

ii) If the ball is let to be more charged, what would be the maximum charge of ball's surface without breaking down of the air around it? (the dielectric strength of air is $3\text{x}10^6$ N/C).

$V_{max} = E_{air}\, R$

$k\, q_{max}/ R = E_{air}\, R$

$q_{max} = E_{air}\, R^2/ k$

to find R, $1800 = 9\ \text{x}10^9\ \text{x}\ 1.8\ \text{x}10^{-8}/ (R+0.06)$

$R = 0.03$ m

$q_{max} = 3\text{x}10^6\ \text{x}\ (0.03)^2/ 9\ \text{x}10^9$

$= 3\text{x}10^{-7}$ C

c) A parallel-plates capacitor in air has a plate separation of 0.5 cm and a plate area of 50 cm^2. The plates are charged to a potential difference of 200 V and disconnected from the source. Then Teflon of dielectric constant 2.5 is inserted and filling the space between the plates. Determine: i) The charge on the plates before and after filling of Teflon.

$C_{before} = \varepsilon_o\, A/ d$

$= 8.85\text{x}10^{-12}\ \text{x}\ 50\text{x}10^{-4} / 0.5\text{x}10^{-2}$

$= 8.85\text{x}10^{-12}$ f

$C_{before} = Q_{before} / V_{before}$

$Q_{before} = 1.77\text{x}10^{-9}\ C = Q_{after}$

ii) The change in energy of the capacitor.

$W = ½\, Q^2/C$

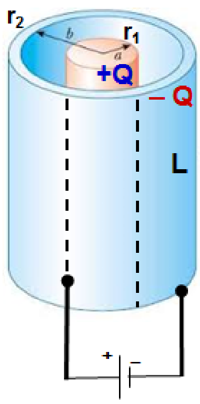
$\Delta W = ½\, Q^2_{before} / C_{before} - ½\, Q^2_{after} / C_{after}$

$= ½\, (1.77\text{x}10^{-9})^2 / 8.85\text{x}10^{-12} - ½\, (1.77\text{x}10^{-9})^2 / 2.5\ \text{x}\ 8.85\text{x}10^{-12} = 1.062\text{x}10^{-7}$ J

<u>Question (3) (16 marks)</u>

a) Derive an expression of cylindrical capacitor consisting of a solid cylindrical conductor of length L, radius r_1 and charge Q is coaxial with a cylindrical thin shell of equal length, radius r_2 and charge – Q?

The capacitance of cylindrical capacitor



$$V_a - V_b = \int_a^b E\ dr$$

$$V_{ab} = \int_{r_1}^{r_2} E\ dr$$

$$\Rightarrow V_{ab} = \int_{r_1}^{r_2} \frac{1}{2\pi\varepsilon_o} \frac{\lambda}{r}\ dr$$

$$\Rightarrow V_{ab} = \frac{\lambda}{2\pi\varepsilon_o} \int_{r_1}^{r_2} \frac{dr}{r} \quad \Rightarrow V_{ab} = \frac{\lambda}{2\pi\varepsilon_o} [\ln r]_{r_1}^{r_2}$$

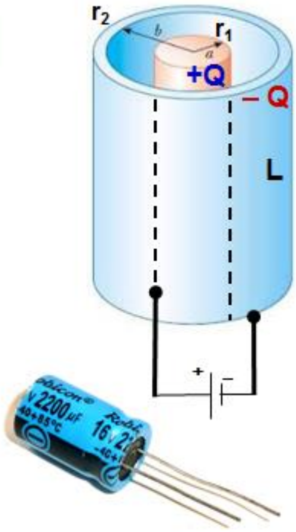
$$\Rightarrow \; V_{ab} = \frac{\lambda}{2\pi\varepsilon_o}\left[\ln r_2 - \ln r_1\right]$$

$$\Rightarrow \; V_{ab} = \frac{Q/L}{2\pi\varepsilon_o}\ln(r_2/r_1)$$

$$\Rightarrow \; V_{ab} = \frac{Q}{2\pi\varepsilon_o L}\ln(r_2/r_1)$$

$$\Rightarrow \; \frac{Q}{V_{ab}} = 2\pi\varepsilon_o \frac{L}{\ln(r_2/r_1)}$$

$$\Rightarrow \; C = 2\pi\varepsilon_o \frac{L}{\ln(r_2/r_1)}$$



b) Consider you have an air condition, a heater and a toaster rated at 2 KW, 1800 W and 600 W respectively are connected in parallel to 220 V input source. i) Find the current density in the input wire if it has a radius of 6 mm?

P1 = 2000 W

P2 = 1800 W

P3 = 600 W

V = 220 V

P = I V

$P_{total} = I_{total} / V$

$I_{total} = (2000+1800+600) / 220$

$= 20$ A

$J = I_{total} / A$

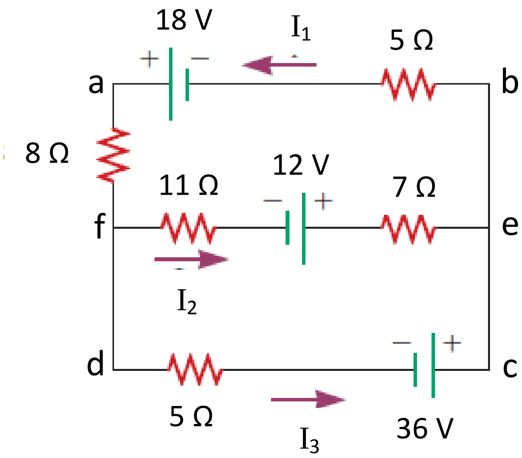
$= 20/ [3.14x (0.006)^2] = 1.77x10^5$ A/m^2

ii) If the cost of electricity is 0.5 Pounds per KW.h, calculate the monthly cost for the air condition if it works 5 hours each day?

W = 2000 x 5 x 30 = 300 KW.h

Electricity cost = 0.5 x 300 = 150 pounds

c) In the opposite circuit, the potential at point (b) is taken to be zero. Find: i) The currents I_1, I_2 and I_3?



$$I1 = I2 + I3 \quad \ldots\ldots (1)$$

Loop febd, $18 + 12 = 11\ I2 + 7\ I2 + 5\ I1 + 8\ I1$

$$13\ I1 + 18\ I2 = 30 \quad \ldots\ldots. (2)$$

Loop fdcef, $36 - 12 = 5\ I3 - 7\ I2 - 11\ I2$

$$5\ I3 - 18\ I2 = 24 \quad \ldots\ldots.. (3)$$

$$I1 = 2.884\ A$$

$$I2 = -\ o.416\ A$$

$$I3 = 3.302\ A$$

ii) The potential of point (F)?

from b to f, $Vb - Vf + 18 = 13\ I1$

$$0 - Vf + 18 = 13\ (2.884)$$

$$Vf = -\ 19.492\ V$$

Model Answer

Question (5) (15 Marks)

5-a) [5 Marks]

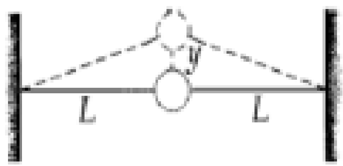
In general , a body is said to move or vibrate with simple harmonic motion if it satisfies the following conditions:

i- its acceleration is always be directed towards a certain fixed point known as the point of reference

ii- Acceleration is proportional to its distance from that point

5-b) [5 Marks]

$$\sum F = -2T\sin\theta \qquad \text{where } \theta = \tan^{-1}\left(\frac{y}{L}\right)$$

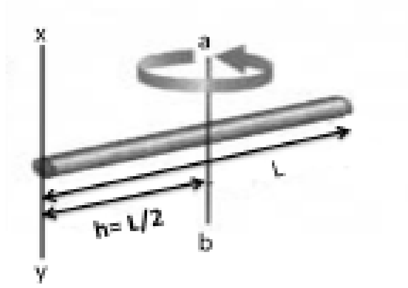


Therefore, for a small displacement

$$\sin\theta \approx \tan\theta = \frac{y}{L} \qquad \text{and} \qquad \boxed{\sum F = \frac{-2Ty}{L}}$$

5-c) [5 Marks]

Moment of inertia of a long thin rod about an axis through the center parallel to one edge



From the parallel axis theory

$$I_{xy} = I_{ab} + Mh^2$$

$$I_{ab} = I_{xy} - Mh^2 \qquad \text{where } I_{xy} = \frac{1}{3}ML^2$$

At one end

$$I_{ab} = \frac{1}{3}ML^2 - M\left(\frac{L}{2}\right)^2$$

$$I_{ab} = \frac{1}{3}ML^2 - \frac{1}{4}ML^2$$

$$I_{ab} = \frac{1}{12}ML^2$$

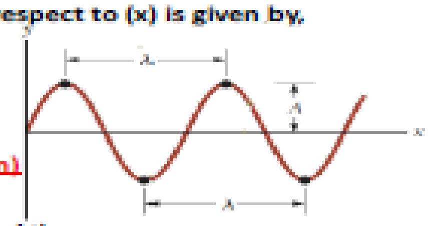
Question (6) (20 Marks)

6-a) [7 Marks]

General Expression of Sinusoidal Wave

- **At t=0 the vertical position (y) of the wave with respect to (x) is given by, since the wave is sinusoidal wave ,**

$$\therefore y(x) = A\sin(\frac{2\pi}{\lambda}x)$$



- If the wave moves to the right(positive x direction) with a speed (v)at some later time (t)

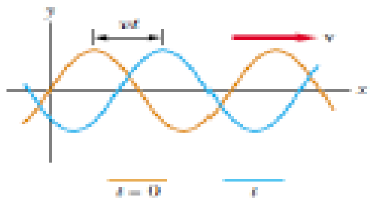
The vertical position (y) of the wave with respect to (x and t) is given by

$$y(x,t) = A\sin[\frac{2\pi}{\lambda}(x - vt)]$$

$$y(x,t) = A\sin[\frac{2\pi}{\lambda}x - \frac{2\pi}{\lambda}vt)]$$

$$y(x,t) = A\sin[\frac{2\pi}{\lambda}x - 2\pi ft)]$$

$$\therefore y(x,t) = A\sin(kx - \omega t)$$



The general expression for sinusoidal wave is:

$$y(x,t) = A\sin(kx - \omega t + \varphi)$$

right(positive x direction)

where A is the amplitude
λ is the wavelength.
f is the frequency
K is the angular wavenumber $k = \frac{2\pi}{\lambda}$ rad/m
φ is the phase constant
ω is the angular frequency $\omega = 2\pi f$ rad/sec.
v is the speed of the sinusoidal wave $v = \lambda f = \frac{\lambda}{T} = \frac{\omega}{k}$
y vertical position

$$y(x,t) = A\sin(kx + \omega t + \varphi)$$

left(negative x direction)

6-b) [7 Marks]

$$v = \sqrt{\frac{T}{\mu}}$$

$$T = \mu v^2 = \rho A v^2 = \rho \pi r^2 v^2$$

$$T = (8\,920\ \text{kg/m}^3)(\pi)(7.50 \times 10^{-4}\ \text{m})^2(200\ \text{m/s})^2$$

$$T = \boxed{631\ \text{N}}$$

6-c) **[6 Marks]**

i- Compare the given wave function with the general form

$$y = (8cm)\sin(3x)\cos(2t) = (2A_o)\sin(kx)\cos(wt)$$

$\therefore k = 3$ rad/cm .

$\therefore \omega = 2$ rad/sec.

Therefore

- $k = \frac{2\pi}{\lambda} = 3$ rad/cm $\quad \therefore \lambda = \frac{2\pi \text{ rad}}{3 \text{ rad/cm}} = 2.09$ cm

- $\omega = 2\pi f = 2$ rad/sec. $\quad f = \frac{w}{2\pi} = \frac{2 \ rad/\sec}{2\pi \ rad} = 0.318Hz$

- $v = \lambda f = \frac{\omega}{k} = \frac{2}{3} = 0.66 \ cm/\sec$

ii- we obtain the amplitude of the simple harmonic motion of the element at the position x = 2.3cm by evaluating the coefficient of the cosine function at this position:

$$y_{max} = (8 \text{ cm})\sin 3x|_{x=2.3}$$

$$= (8 \text{ cm}) \sin (6.9 \text{ rad}) = 4.6 \text{ cm}$$