Benha University	**Final Term Exam**
Faculty of Engineering- Shoubra	**Date: 14th of june 2014**
Electrical Engineering Department	**Mathematics (2-B) -EMP 182**
1st Year Communication تخلفات	**Duration : 3 hours**

- Answer all the following questions
- Illustrate your answers with sketches when necessary.
- The exam. Consists of one page
- No. of questions:5
- Total Mark: 80 Marks

Q1) Evaluate the following integrals: **[25]**

$$\int_0^\infty \sqrt[4]{x}\, e^{-x^3} dx, \quad \oint_{|z|=4} \frac{\cos(z)}{z^2-6z+5} dz, \quad \int_2^\infty e^{-x^2+4x-4} dx, \quad \int_0^3 \frac{t^2\, dt}{\sqrt{3-t}}, \quad \int_0^\infty \left(\frac{e^{2t}-\cos 3t}{t}\right) dt$$

Q2) Find v(x,y), such that f(z) = u(x,y) + i v(x,y) is analytic, where $u(x,y)= e^x(x\cos y - y\sin y)$, then express f(z) in terms of z. **[10]**

Q3-a) Find Laplace transform for the following functions: **[10]**

i) $f(t) = \cos 2t \cosh 3t + \dfrac{\cos 2t - \cos 3t}{t}$, ii) $g(t) = \begin{Bmatrix} t & 0 < t \le 2 \\ 2 & 2 < t \le 3 \\ t-1 & t > 3 \end{Bmatrix} + (t-3)^2\, U(t-2)$

b) Find inverse Laplace for the following functions: **[10]**

$$F(s) = \frac{s\, e^{-2s}}{s^2+6s+20} + \frac{1}{(3s+4)^3}, \qquad G(s) = \frac{1}{s^3(s^2+9)} + \frac{3s+2}{s^2+6s+5}$$

Q4) Evaluate the following integrals: **[15]**

i) $\oint_c \dfrac{\cos(z)}{z^2-6z+5} dz$, where c is the circle |z| = 4.

ii) $\oint_c \dfrac{z^3+5z+7}{(z-i)^2} dz$, c: | z-2| + | z+2| = 6,

iii) $\oint_c \dfrac{dz}{(z^2+9)^2}$, using Cauchy's residue theorem c: |z+1-i| = 3.

Q5) Solve the following differential equations using Laplace Transform: **[10]**

i) $3y' + 4y = e^{2t}$, $y(0) = 1/3$, ii) $y'' + y = 2t$, $y(0) = 3$, $y'(0) = 1$

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Model answer

Answer of Q1

z = 5 is outside contour, while z = 1 is inside contour, therefore

$$\oint_c \frac{\cos(z)}{z^2-6z+5}\,dz = \oint_c \frac{\cos(z)/(z-5)}{z-1}\,dz = -\frac{1}{2}\pi i \cos(1)$$

$$\int_2^\infty e^{-x^2+4x-4}\,dx = \int_2^\infty e^{-(x-2)^2}\,dx$$, Put $y = (x-2)^2 \Rightarrow dy = 2(x\text{-}2)\,dx \Rightarrow dx = \frac{dy}{2\sqrt{y}}$,

therefore $\int_2^\infty e^{-(x-2)^2}\,dx = \frac{1}{2}\int_0^\infty y^{-1/2}\,e^{-y}\,dy = \frac{\sqrt{\pi}}{2}$

$\int_0^3 \frac{t^2\,dt}{\sqrt{3-t}} = \frac{27}{\sqrt{3}}\beta(m,n)$, where m-1 = 2 and n-1 = -½, i.e. m = 3 and n = ½

$$\int_0^\infty \left(\frac{e^{2t}-\cos 3t}{t}\right)e^{-3t}\,dt = L\left\{\frac{e^{2t}-\cos 3t}{t}\right\}_{s=3} = \int_s^\infty \left(\frac{1}{s-2}-\frac{s}{s^2+9}\right)ds\;_{s=3} = \ln\left[\frac{\sqrt{s^2+9}}{s-2}\right]_{s=3}$$
$$=\frac{1}{2}\ln 18$$

Answer of Q2

$u_x = e^x(x\cos y - y\sin y + \cos y)$, $u_y = e^x(-x\sin y - \sin y - y\cos y)$, since f(z) is analytic, therefore f(z) satisfy Cauchy Riemann such that $u_x = v_y = e^x(x\cos y - y\sin y + \cos y)$, thus by integration with respect to y, we get $v(x,y) = e^x(x\sin y + y\cos y) + \phi(x)$, thus:
$v_x = e^x(x\sin y + \sin y + y\cos y) + \phi`(x) = -u_y = -e^x(-x\sin y - \sin y - y\cos y)$, therefore $\phi`(x) = 0$, hence $\phi(x) = c$, thus $v(x,y) = e^x(x\sin y + y\cos y) + c$. Put y=0, x = z , we get $f(z) = z\,e^z$.

Answer of Q3

$$L\{\cos 2t \cosh 3t\} = L\{\cos 2t(e^{3t} + e^{-3t})/2\} = \frac{1}{2}\left[\frac{s-3}{(s-3)^2+4}+\frac{s+3}{(s+3)^2+4}\right].$$

$$L\left\{\frac{\cos 2t-\cos 3t}{t}\right\} = \int_s^\infty \left(\frac{s}{s^2+4}-\frac{s}{s^2+9}\right)ds = \frac{1}{2}\ln\left[\frac{s^2+9}{s^2+4}\right]$$

Since $g(t) = \begin{cases} t & 0 < t \le 2 \\ 2 & 2 < t \le 3 \\ t-1 & t > 3 \end{cases}$ = t[u(t) – u(t-2)] + 2[u(t-2) –u(t-3)]+ (t-1)u(t-3)= tu(t) – (t-2)u(t-2) + (t-3)u(t-3), therefore $G(s) = 1/s^2 - (1/s^2)e^{-2s} + (1/s^2)e^{-3s}$

$h(t) = (t-3)^2\ U(t-2) = (t-2\ -1)^2\ U(t-2) = (t-2)^2\ U(t-2) - 2(t-2)\ U(t-2) + U(t-2)$, therefore $H(S) = (2/s^3)e^{-2s} - (1/s^2)e^{-2s} + (1/s)\ e^{-2s}$

b)

$$F(s) = \frac{s\,e^{-2s}}{s^2+6s+20} + \frac{1}{(3s+4)^3} = \frac{(s+3-3)\,e^{-2s}}{(s+3)^2+11} + \frac{1}{27(s+4/3)^3}$$, therefore

$$f(t) = e^{-3(t-2)}[\cos\sqrt{11}(t-2) - \frac{3}{\sqrt{11}}\sin\sqrt{11}(t-2)]U(t-2) + \frac{1}{54}e^{-(4/3)t}t^2$$

$$G(s) = \frac{1}{s^3(s^2+9)} + \frac{3s+2}{s^2+6s+5} = \frac{1}{s^3(s^2+9)} + \frac{3(s+3-3)+2}{(s+3)^2-4}$$, therefore

$$g(t) = \frac{1}{9}\int_{u=0}^{t}(u - \frac{\sin 3u}{3})\ du + 3e^{-3t}\cosh 2t - \frac{7}{2}e^{-3t}\sinh 2t = \frac{1}{9}[\frac{t^2}{2} + \frac{\cos 3t}{9}] + 3e^{-3t}\cosh 2t - \frac{7}{2}e^{-3t}\sinh 2t$$

Answer of Q4

i- z = 5 is outside contour, while z = 1 is inside contour, therefore

$$\oint_c \frac{\cos(z)}{z^2-6z+5}dz = \oint_c \frac{\cos(z)/(z-5)}{z-1}dz = -\frac{1}{2}\pi i\cos(1)$$

ii- z = i is inside contour, therefore $\oint_c \frac{z^3+5z+7}{(z-i)^2}dz = 4\pi i$

iii- Since z = 3i is inside contour, while z = -3i is outside contour, therefore

$\oint_c \frac{dz}{(z^2+9)^2} = 2\pi i\ Res_{z=3i}$, where $Res_{z=3i} = -(1/108)i$, therefore $\oint_c \frac{dz}{(z^2+9)^2} = \pi/54$

Answer of Q5

By taking Laplace for Both equations , therefore

i) $3[\,sY(s) - y(0)] + 4\,Y(s) = 1/(s-2)$, therefore $Y(s) = \frac{1}{(3s+4)(s-2)} + \frac{1}{(3s+4)}$, thus

$$Y(s) = \frac{1}{3[s^2-(2/3)s-(8/3)]} + \frac{1}{3[s+(4/3)]} = \frac{1}{3\,([s-(1/3)]^2-(25/9))} + \frac{1}{3[s+(4/3)]},$$

therefore $y(t) = \frac{1}{5}\,e^{(1/3)t}\sinh(5/3)t + \frac{1}{3}\,e^{(-4/3)t}$

ii) $(s^2+1)\,Y(s) - 3s - 1 = \frac{2}{s^2}$, therefore $Y(s) = \frac{2}{s^2(s^2+1)} + \frac{3s+1}{(s^2+1)}$, therefore

$$y(t) = \int_{u=0}^{t} (1-\cos u)\,du + 3\cos t + \sin t = 2[t - \sin t] + 3\cos t + \sin t$$