Benha University Faculty of Engineering- Shoubra Electrical Engineering Department Second year communication		Final Term Exam Date: 4/6/ 2014 Engineering Mathematics (3)B Duration : **3 hours**
Answer all the following questions	No. of questions : **Two**	Total Mark: **80**

Question 1 **[40 marks]**

(a) Show that the iteration formula $x_{n+1} = \frac{2x_n^3 + A}{3x_n^2}; n = 0,1,2,...$ is used **[10 marks]** to find cubic root of real number **A** ,hence use the formula to find cubic root of **30**

(b) Find cubic interpolation polynomial which interpolate the function y= **f(x)** at the points (1,-4) , (2,8) , (5,140) , (8,542) , (9,764) , (10,1040) .Hence find the value of x which make **f(x) = 0** by fixed method . **[10 marks]**

(c) By using Euler's method solve the I.V.P $y' - 1 = xy ; y(0) = 1$ **[10 marks]** to get $y(0.6)$ with $h = 0.2$

(d) Find the deflection u(x ,t) of the vibrating string (length L = π) ,ends fixed, and $c^2 = 1$ corresponding to zero initial velocity and initial deflection **f(x)= sin2x** **[10 marks]**

Question 2 **[40 marks]**

(a) Given the heat equation $\frac{\partial u}{\partial t} = 2\frac{\partial^2 u}{\partial x^2}; t \succ 0$ **[20 marks]**

subject to $u(0,t) = 0, u(3,t) = 0$ and $u(x,0) = x(x-3)$

(i) Find its Exact solution using separation method and Fourier series

(ii) Using the finite difference scheme to compute **u (x =1 , t = 1)** take h= 0.5 ; k =0. 5

(b) Find $f'(1.005)$ and $f'(1.015)$ for the following data **[10 marks]**

x	1.00	1.01	1.02
y	1.27	1.32	1.38

And find a root for the same data by inverse Lagrange interpolation

(c) Deduce the Exact solution of the wave equation **[10 marks]**

Good Luck

Dr.Eng. Zaki Ahmed Zaki

Second year comm.   Final term Exam (May 2014)

Model Answer

Q1

(a)

$$x^3 - A = 0; x_{n+1} = x_n - [\frac{x_n^3 - A}{3x_n^2}] = \frac{2x_n^3 + A}{3x_n^2}; n = 0,1,2,...$$

$$x_0 = \frac{3+4}{2} = 3.5, \boxed{Root = 3.107}$$

(b)

$$f(x) = -4 + 12(x-1) + (x-1)(x-2)8 + (x-1)(x-2)(x-5)7 = \boxed{x^3 + 5x - 10}$$

$$\text{fixed is}: x^3 = 10 - 5x; x_{n+1} = (10 - 5x_n)^{\frac{1}{3}}; \boxed{Root = 1.4235}$$

x	y			
1	-4	12	8	
2	8	44	15	1
5	140	134		
8	542			

(C)

$$y' - 1 = xy; y_{i+1} = y_i + 0.2(1 + x_i y_i), x_0 = 0, y_0 = 1$$

$$y_1 = y_0 + 0.2(1 + x_0 y_0) = 1.2, y_2 = 1.448, y_3 = \boxed{y(0.6) = 1.7638}$$

(d)Wave Equation

$$u_{tt} = c^2 u_{xx}, u(0,t) = u(L,t) = 0, u(x,0) = f(x) = \sin 2x, u_t(x,0) = g(x) = 0$$

$$u(x,t) = \sum_{n=1}^{\infty} \sin\frac{n\pi}{L}x\,(A_n \cos\frac{cn\pi}{L}t + B_n \sin\frac{cn\pi}{L}t); A_n = \frac{2}{L}\int_0^L f(x)\sin\frac{n\pi}{L}x\,dx;$$

$$B_n = \frac{2}{cn\pi}\int_0^L g(x)\sin\frac{n\pi}{L}x\,dx; \rightarrow \boxed{B_n = 0}$$

Q2 (a)

$$u(x,t)=F_{(x)}G_{(t)};FG'=2F''G;\frac{F''}{F}=\frac{G'}{2G}=-\alpha^2$$

$$F=c_1\cos\alpha x+c_2\sin\alpha x;G=Ae^{-2\alpha^2 t}$$

$$u(x,t)=e^{-2\alpha^2 t}[B\cos\alpha x+D\sin\alpha x];u(0,t)=0\,then\,B=0,$$

$$u(3,t)=D\sin 3\alpha e^{-2\alpha^2 t}=0,\alpha=\frac{n\pi}{3},n=1,2,3,...$$

$$\boxed{u(x,t)=\sum_{n=1}^{\infty}D_n e^{\frac{-2n^2\pi^2 t}{9}}\sin\frac{n\pi}{3}x, and\, D_n=\frac{2}{3}\int_0^3 x(x-3)\sin\frac{n\pi}{3}x\,dx}$$

(b)

$$r=\frac{ka}{h^2}=\frac{0.5(2)}{(0.5)^2}=4;u_{i,j+1}=4(u_{i+1,j}+u_{i-1,j})-7u_{i,j}$$

$$u_1=0.75,u_2=0,u_3=-0.25,\boxed{u_9=u(x=1,t=1)=2}$$

(c)

$$f'(1.005)=\frac{1.32-1.27}{2(0.005)}=\boxed{5};f'(1.015)=\frac{1.38-1.32}{2(0.005)}=\boxed{6}$$

$$x(y)=\frac{(y-1.32)(y-1.38)}{(1.27-1.32)(1.27-1.38)}(1)+\frac{(y-1.27)(y-1.38)}{(1.32-1.27)(1.32-1.38)}(1.01)+\frac{(y-1.27)(y-1.32)}{(1.38-1.27)(1.38-1.32)}(1.02)$$

$$let\, y=0\rightarrow \boxed{Root=0.238}$$

(d) **Wave Equation**

$$u_{tt}=c^2u_{xx},u(0,t)=u(L,t)=0,u(x,0)=f(x),u_t(x,0)=g(x)$$

$$u(x,t)=F_{(x)}G_{(t)};$$

$$FG''=c^2F''G;\frac{F''}{F}=\frac{G''}{c^2G}=-\alpha^2,$$

$$\boxed{F_{(x)}=\sum_{n=1}^{\infty}c_n\sin\frac{cn\pi}{L}x,}$$

$$\boxed{G_{(t)}=c_3\cos\frac{cn\pi}{L}t+c_4\sin\frac{cn\pi}{L}t}$$

$$u(x,t)=\sum_{n=1}^{\infty}\sin\frac{n\pi}{L}x(A_n\cos\frac{cn\pi}{L}t+B_n\sin\frac{cn\pi}{L}t);$$

$$A_n=\frac{2}{L}\int_0^L f(x)\sin\frac{n\pi}{L}x\,dx;B_n=\frac{2}{cn\pi}\int_0^L g(x)\sin\frac{n\pi}{L}x\,dx;$$