Benha University Faculty of Engineering- Shoubra Eng. Mathematics & Physics Department Preparatory Year (تخلفات)		Final Term Exam Date: May 23, 2016 Course: Mathematics 1 – B Duration: **3** hours

- The Exam consists of one page — Answer **All** Questions
- No. of questions: 4 — Total Mark: 100

Question 1

(a)If $A = \begin{bmatrix} 1 & 3 & 1 \\ 2 & 0 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 1 & 4 \\ 3 & 3 & 0 \end{bmatrix}$. Find, if possible : A + B, A.B, A.B^t, |A|, |A`.B|. 10

(b)If $A = \begin{bmatrix} -1 & 2 \\ 2 & 2 \end{bmatrix}$.

(i)Find the eigenvalues and eigenvectors of A. 8

(ii)Write Hamilton equation and find A^{-1}. 5

(iii)Find the eigenvalues of : $f(A) = 2^A$ 2

Question 2

(a)Using the binomial theorem, expand : $\frac{1}{\sqrt{3x+4}}$ 4

(b) Solve the linear system: 3x + 2y – 3z = 4, 2x – y = 1, x + 3y – 3z = 5. 5

(c) Find S , S_{10} from each series: (i) $\sum_{r=1}^{n}(2r-1)(1+r^2)$ (ii) $\sum_{r=1}^{n}\frac{1}{r^2+3r+2}$ 8

(d)Using the mathematical induction, prove that:

(i) $1+3+5+\cdots+(2n-1)=n^2$ (ii) $1+2+4+\cdots+2^{n-1}=2^n-1$ 8

Question 3

(a)Transform the equation $x^2-6xy+9y^2+4x+8y+15=0$ to new axes through (– 2, 1). 6

(b)A point moves so that its distance from the x –axis is half of its distance from the point (2, 3). Find the equation of its locus. 6

(c)Show that the equation : $12x^2+7xy-10y^2+13x+45y-35=0$ represents two straight lines and find the angle between them. 6

(d)Find the equation of the circle which passes through the points (1, 2), (8, 9) and cuts the circle $x^2+y^2=25$ at right angle. 7

Question 4

(a)Find the vertex , focus, directrix and length of latus rectum the parabola : $y^2+4x+2y-8=0$. 6

(b)Find the vertices, foci and directrix of the ellipse : $4x^2+9y^2=144$. 6

(c)Show that the line 2x + y = 4 is tangent to the circle : $x^2+y^2+6x-10y+29=0$. 6

(d)Trace the curve : $9x^2-24xy+16y^2-18x-101y+19=0$. 7

Good Luck — *Dr. Mohamed Eid* — *Dr. Fathi Abdsallam*

Answer

Question 1

(a) A + B = $\begin{bmatrix} 3 & 4 & 5 \\ 5 & 3 & 2 \end{bmatrix}$, A.B and |A| are not exist.

$A.B` = \begin{bmatrix} 1 & 3 & 1 \\ 2 & 0 & 2 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 1 & 3 \\ 4 & 0 \end{bmatrix} = \begin{bmatrix} 9 & 12 \\ 12 & 6 \end{bmatrix}.$

$A`.B = \begin{bmatrix} 1 & 2 \\ 3 & 0 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 2 & 1 & 4 \\ 3 & 3 & 0 \end{bmatrix} = \begin{bmatrix} 8 & 7 & 4 \\ 6 & 3 & 12 \\ 8 & 7 & 4 \end{bmatrix}.$  |A`.B| = 0

-- ***(10 Marks)***

(b)(i) $|A - \lambda I| = \begin{vmatrix} -1-\lambda & 2 \\ 2 & 2-\lambda \end{vmatrix} = (-1-\lambda)(2-\lambda) - 4 = \lambda^2 - \lambda - 6 = 0$

Then, the eigenvalues are: $\lambda_1 = 3, \quad \lambda_2 = -2.$

From the equation, $\begin{bmatrix} -1-\lambda & 2 \\ 2 & 2-\lambda \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$

For : $\lambda_1 = 3$, $\begin{bmatrix} -4 & 2 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$

Then – 4x + 2y = 0, 2x – y = 0

Then 2x = y = any number except 0

Put x = 1, we get y = 2 and the

corresponding eigenvector is: $X_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

For : $\lambda_2 = -2$, $\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$

Then x + 2y = 0, 2x + 4y = 0

Then x = – 2y = any number except 0

Put y = 1, we get x = – 2 and the

corresponding eigenvector is: $X_2 = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$

-- ***(8 Marks)***

(b)(ii)The Hamilton equation is: $A^2 - A - 6I = 0$

Since |A| = – 6. Then inverse A exists. Multiply the Hamilton equations by A^{-1}.

Then $A - I - 6A^{-1} = 0$.

Then $A^{-1} = \frac{1}{6}(A - I) = \frac{1}{6}\left(\begin{bmatrix} -1 & 2 \\ 2 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right) = \frac{1}{6}\begin{bmatrix} -2 & 2 \\ 2 & 1 \end{bmatrix}$

-- ***(5 Marks)***

(b)(iii)The eigenvalues of $f(A) = 2^A$ are $f(3) = 2^3 = 8$, $f(-2) = 2^{-2} = \frac{1}{4}$.

-- ***(2 Marks)***

Question 2

(a) $\frac{1}{\sqrt{3x+4}} = \frac{1}{2}(1 + \frac{3}{4}x)^{-\frac{1}{2}} = \frac{1}{2}\left(1 - \frac{3}{8}x + \frac{27}{8 \text{ x} 16}x^2 + \cdots\right), \quad |\frac{3}{4}x| < 1$

-- ***(4 Marks)***

(b) $G = \left[\begin{array}{ccc:c} 1 & 3 & -3 & 5 \\ 3 & 2 & -3 & 4 \\ 2 & -1 & 0 & 1 \end{array}\right] = \left[\begin{array}{ccc:c} 1 & 3 & -3 & 5 \\ 0 & -7 & 6 & -11 \\ 0 & -7 & 6 & -9 \end{array}\right] = \left[\begin{array}{ccc:c} 1 & 3 & -3 & 5 \\ 0 & -7 & 6 & -11 \\ 0 & 0 & 0 & 2 \end{array}\right]$

We see that rank A is 2 but rank G is 3. Then, there is no solution.

-- ***(5 Marks)***

(c)(i)Since $u_r = (2\mathrm{r}-1)(1+\mathrm{r}^2) = 2\mathrm{r}^3 - \mathrm{r}^2 + 2\mathrm{r} - 1$

Then $S_n = \frac{1}{2}\mathrm{n}^2(\mathrm{n}+1)^2 - \frac{1}{6}\mathrm{n}(\mathrm{n}+1)(2\mathrm{n}+1) + \mathrm{n}(\mathrm{n}+1) - \mathrm{n}$

Then $S = \infty$

$$S_0 = (50)(121) - \frac{1}{6}(10)(11)(21) + 100 = 5765$$

-- ***(4 Marks)***

(c)(ii)Since $u_r = \frac{3}{\mathrm{r}^2+3\mathrm{r}+2} = \frac{1}{\mathrm{r}+1} - \frac{1}{\mathrm{r}+2}$. Then $S_n = \frac{1}{2} - \frac{1}{\mathrm{n}+2}$.

Then $S = \frac{1}{2}$ and $S_{10} = \frac{1}{2} - \frac{1}{12} = \frac{5}{12}$

-- ***(4 Marks)***

(d)(i)Prove that: $1 + 3 + 5 + \cdots + 2\mathrm{n} - 1 = \mathrm{n}^2$

(1)At n = 1, the left hand side (L.H.S) of this relation is 1 and the right hand side (R.H.S) of this relation is 1. Then this relation is true at n = 1.

(2)Assume that this relation is true at n = k.

This means that $1 + 3 + 5 + \cdots + 2\mathrm{k} - 1 = \mathrm{k}^2$

(3)We shall prove that this relation is true at n = k + 1.

Or prove that, $1 + 3 + 5 + \cdots + (2\mathrm{k}+1) = (\mathrm{k}+1)^2$

From step 2, add 2k + 1 to both sides, we get

$$1 + 3 + 5 + \cdots + (2\mathrm{k}-1) + (2\mathrm{k}+1) = \mathrm{k}^2 + (2\mathrm{k}+1) = (\mathrm{k}+1)^2$$

Then this relation is true for all n.

-- ***(4 Marks)***

(d)(ii) prove that: $1 + 2 + 4 + \cdots + 2^{\mathrm{n}-1} = 2^{\mathrm{n}} - 1$

(1)At n = 1, the left hand side (L.H.S) of this relation is 1 and the right hand side (R.H.S) of this relation is 1. Then this relation is true at n = 1.

(2)Assume that this relation is true at n = k.

This means that $1 + 2 + 4 + \cdots + 2^{\mathrm{k}-1} = 2^{\mathrm{k}} - 1$

(3)We shall prove that this relation is true at $n = k + 1$.

Or prove that, $1 + 2 + 4 + \cdots + 2^{k} = 2^{k+1} - 1$

From step 2, add 2^{k} to both sides, we get

$$1 + 2 + 4 + \cdots + 2^{k-1} + 2^{k} = 2^{k} - 1 + 2^{k} = 22^{k} - 1 = 2^{k+1} - 1$$

Then this relation is true for all n.

-- ***(4 Marks)***

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Question (3)

(a)Transform the equation $x^2 - 6xy + 9y^2 + 4x + 8y + 15 = 0$ to new axes through $(-2,1)$.

Answer

Put $x = x - 2$ and $y = y + 1$

$(x - 2)^2 - 6(x - 2)(y + 1) + 9(y + 1)^2 + 4(x - 2) + 8(y + 1) + 15 = 0$

$\boxed{x^2 - 6xy + 9y^2 - 6x + 38y + 57 = 0}$

(b)A point moves so that its distance from the x $-$axis is half of its distance from the point $(2,3)$ .Find the equation of its locus.

Answer

Let the point is $P(x, y)$ itsdistancefromthe x $-$axis is y and its distancefromthepoint $(2,3)$ is $\sqrt{(x-2)^2 + (y-3)^2}$ the locus described by

$y = \frac{1}{2}\sqrt{(x-2)^2 + (y-3)^2}$

$4y^2 = x^2 - 4x + 4 + y^2 - 6x + 9$

$\boxed{3y^2 - x^2 + 4x + 6y - 13 = 0}$

(c)Show that the equation $12x^2 + 7xy - 10y^2 + 13x + 45y - 35 = 0$ represents to two straight lines, find the angle between them, and find separately the two straight lines.

Answer

The general form for equation which represents to two straight lines is

$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ by comparison we find

$a = 12,\ \ 2h = 7,\ \ b = -10,\ \ 2g = 13, 2f = 45,\ \ c = -35$

the discriminant is

$$\Delta = \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = \begin{vmatrix} 12 & \frac{7}{2} & \frac{13}{2} \\ \frac{7}{2} & -10 & \frac{45}{2} \\ \frac{13}{2} & \frac{45}{2} & -35 \end{vmatrix} = \frac{1}{8}\begin{vmatrix} 24 & 7 & 13 \\ 7 & -20 & 45 \\ 13 & 45 & -70 \end{vmatrix} = 0$$

hence the given equation represents to two straight lines

since $12x^2 + 7xy - 10y^2 = (3x - 2y)(4x + 5y)$ then

$$12x^2 + 7xy - 10y^2 + 13x + 45y - 35 = (3x - 2y + c_1)(4x + 5y + c_2)$$

$$= 12x^2 + 7xy - 10y^2 + (3c_2 + 4c_1)x + (5yc_1 - 2c_2)y + c_1c_2 = 0$$

Then $(4c_1 + 3c_2) = 13$, $(5yc_1 - 2c_2) = 45$ and $c_1c_2 = 35$

solve the first two equation we get $c_1 = 7,\ c_2 = -5$

Therefore the required separated equations of the lines are

$3x - 2y + 7 = 0$ and $4x + 5y - 5 = 0$ it is noticed that the two lines

$3x - 2y = 0$ and $3x - 2y + c_1 = 0$ are parallel . Also

$4x + 5y = 0$ *and* $4x + 5y + c_2 = 0$ are parallel lines.

Hence the angle between the two lines represented by the general equation equal the angle between the two lines represented by the homogenous equation $ax^2 + 2hxy + by^2 = 0$

$$\tan\theta = \pm\frac{2\sqrt{h^2 - ab}}{a + b} = \pm\frac{2\sqrt{(49/4) + 120}}{2} = 11.5 \text{ and } \theta = 85^\circ\ 1'\ 48''$$

(d)Find the equation of the circle which passes through the points (8, 9), (1, 2) and cut the circle $x^2 + y^2 = 25$ at a right angle.

Answer

Let the equation be $x^2 + y^2 + 2gx + 2fy + c = 0$ (1)

Since the circle cut the circle $x^2 + y^2 = 25$ (2)

Then the line segment of the centers and the two radii form aright angle triangle

$g^2 + f^2 = r_1^2 + r_2^2$

$g^2 + f^2 = 25 + g^2 + f^2 - c$

$c = 25$

$(8,9)$ on the circle then $8^2 + 9^2 + 16g + 18f + 25 = 0$

$(1,2)$ on the circle then $1^2 + 2^2 + 2g + 4f + 25 = 0$

Solve to find f , g

Question (4)

(a)Find the vertex, focus, directrix and length of the latus rectum of the parabolas

$y^2 + 4x + 2y - 8 = 0$

Answer

$y^2 + 4x + 2y - 8 = 0$

$(y^2 + 2y + 1) = -4x + 9$

$(y + 1)^2 = -4(x + 2.25) \quad a = -1$

Since a is negative then the curve open to the left side

Vertex at $(-2.25, -1)$

Axis of symmetry are $x = -2.25, y = -1$

Focus at $(-2.25 + a, -1) = (-2.25 - 1, -1) = (-3.25, -1)$

Length of the latus rectum = 4a =4

(b)Findthevetices,foci, directrix of the ellipse $4x^2 + 9y^2 = 144$

Answer

$4x^2 + 9y^2 = 144$. Divide the equation by 144 , we get $\frac{x^2}{36} + \frac{y^2}{16} = 1$

$a^2 = 36, \ b^2 = 16$

The center at (0, 0) and the vertices at (±6.0),(0, ±4)

Major axis = 2a =12 and   minor axis = 2b =8

$$b^2 = a^2(1-e^2)$$

$$e = \sqrt{1-\frac{b^2}{a^2}} = \sqrt{1-\frac{16}{36}} = \sqrt{\frac{20}{36}} = \frac{\sqrt{5}}{3}$$

Foci at $(\pm ae, 0) = (\pm 2\sqrt{5}, 0)$

Equation of the directrix is $x = \pm\frac{a}{e} = \pm\frac{3\times 6}{\sqrt{5}} = \pm\frac{18}{5}\sqrt{5}$

(c)Show that the line $2x + y = 4$ is a tangent to the circle $x^2 + y^2 + 6x - 10y + 29 = 0$.

Answer

The given line is tangent if the perpendicular from the center of the circle equal its radius in length

Center of the circle at (-3, 5) and its radius equal $\sqrt{9+25-29} = \sqrt{5}$

And the perpendicular from the center to the line is $\left|\frac{2(-3)+5-4}{\sqrt{4+1}}\right| = \frac{5}{\sqrt{5}} = \sqrt{5}$ then the line is a tangent.

(d)Trace the curve $9x^2 - 24xy + 16y^2 - 18x - 101y + 19 = 0$

Answer

$$\Delta = \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = \begin{vmatrix} 9 & -12 & -9 \\ -12 & 16 & -101/2 \\ -9 & -101/2 & 19 \end{vmatrix} =$$

$h^2 = (-12)^2 = 144,\ ab = (9)(16) = 144$. Since $h^2 = ab$ The conic section is a parabola

The equation is $(3x-4y)^2 - 18x - 101y + 19 = 0$ and the straight line $\mathbf{3x - 4y = 0}$ as a new axis of x

We rotate the axes by angle $\boldsymbol{\theta}$ such that $\tan\theta = \frac{3}{4}$ substitute in the equation by

$$x = X\cos\theta - Y\sin\theta = \frac{4X - 3Y}{5}$$

$$y = X\sin\theta + Y\cos\theta = \frac{3X + 4Y}{5}$$

The equation becomes $\quad 25Y^2 - 75X - 70Y + 19 = 0$

and complete the square we get $\left(Y - \frac{7}{5}\right)^2 = 3\left(X + \frac{2}{5}\right) \qquad (*)$

take the point $A\left(-\frac{2}{5}, \frac{7}{5}\right)$ as the origin for the new axes OX, OY, and The focus at $F\left(\frac{7}{20}, \frac{7}{5}\right)$

then the equation of the cone with respect to the new coordinates is $Y^2 = 3X$ vertex at A

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Final Exam and ILOs

Course Title: Mathematics 1-**B** **Code:** EMP 021

Questions	ILOs		
	Knowledge and Understanding	Intellectual Skills	Professional and Practical Skills
	a.1	b.1	c.1
Q1	√	√	√
Q2	√	√	
Q3	√		√
Q4	√	√	√

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