Model answer

Final Physics (B) exam

29/5/ 2016

Question (1) (13 marks)

1-a) zeroth law of thermodynamics state that if there is thermal equilibrium between two surfaces A and B and other thermal equilibrium between B and C then there is thermal equilibrium between surfaces A and C.

Thermal equilibrium is defined as the state of matter at which the rate of emission energy per unite area per unite time is equal to the rate of absorption energy per unite area per unite time for the same surface. And at this state the temperature becomes constant.

1-b) Find the relation describe change of temperature in Fahrenheit and the corresponding change of temperature in Kelvin. Calculate change in T by 50 F in Kelvin.

$T_F = \frac{9}{5} T_C + 32 \quad and \quad T_k = T_C + 273$

Apply the two equations for different temperatures 1 and 2 and subtracts each others as

$T_{F2} = \frac{9}{5} T_{C1} + 32 \qquad T_{F2} = \frac{9}{5} T_{C2} + 32 \qquad \Delta T_F = (T_{F2} - T_{F2}) = \frac{9}{5} \Delta T_C$

But $\Delta T_C = \Delta T_k \qquad \therefore \Delta T_F = \frac{9}{5} \Delta T_k$ or $\Delta T_k = \frac{5}{9} \Delta T_F = \frac{5}{9} \times 50\ F = 27.78\ K$

1-c) A 800 gm of CO_2 gas occupy 2 liters at 80 ℃ expanded isobaric to volume 3 liter. Calculate:

1- molar heat capacities at constant volume and at constant pressure and γ -constant

2-initial pressure 3-final pressure and temperature

4- change in root mean square velocity. 5- change in mean kinetic energy of molecule

6- change in entropy. 7- change in internal energy and its sign

8- the mechanical work and which do this work? 9- change in entropy

10- how much heat must supplied or removed

Molecular weight for CO_2=2*16+12=48 gm/mole=0.048 Kgm/mole

∴ number of moles $n = \frac{mass}{molecular\ weight} = \frac{800\ gm}{48 \frac{gm}{mole}} = 16.57\ mole$

1-The carbon dioxide is tri-atomic gas has 7 degree of freedom

The internal energy $u = 7 \times \frac{1}{2} RT = \frac{7}{2} RT$ but the molar heat capacity at constant volume C_V where

$$\therefore C_V = \frac{dU}{dT} = \frac{7}{2} R = \frac{7}{2} \times \frac{8.31 J}{moleK} = 29.085 \frac{J}{moleK}$$

And the molar heat capacity at constant pressure $C_P = C_V + R = \frac{9}{2} R = \frac{9}{2} \times \frac{8.31 J}{moleK} = 37.395 \frac{J}{moleK}$

2- Initial pressure $P = \frac{nRT}{V} = \frac{16.57\ mole \times \frac{8.31J}{moleK} \times (80+273)}{2 \times 10^{-3} m^3} = 243 \times 10^5\ Pa$

3- final pressure at isobaric expansion is the same of initial pressure $243 \times 10^5\ Pa$

The final temperature can calculated from state of constant pressure $\therefore \frac{V_1}{V_2} = \frac{T_1}{T_2}$

$$\therefore\ T_2 = T_1 \times \left(\frac{V_2}{V_1}\right) = (80+273)K \times \left(\frac{3\ litre}{2\ litre}\right) = 529.5\ K = 256.5\ ℃$$

4- Change in root mean square velocity

$$\Delta v_{r.m.s} = v_2 - v_1 = \sqrt{\frac{3RT_2}{M}} - \sqrt{\frac{3RT_1}{M}} = \sqrt{\frac{3R}{M}} \times \left(\sqrt{T_2} - \sqrt{T_1}\right)$$

$$\Delta v_{r.m.s} = \sqrt{\frac{3 \times 8.31}{0.048 \frac{\mathbf{Kgm}}{\mathbf{mole}}}} \times \left(\sqrt{529.5\ K} - \sqrt{353\ k}\right) = 22.7898 \times (23 - 18.788) = 96\ m/s$$

5-Change in kinetic energy of molecules

$$\Delta(K.E)_{molecule} = \frac{3}{2} K(T_2 - T_1) = \frac{3}{2}\frac{R}{N_A}(T_2 - T_1) = \frac{3}{2} \times \frac{8.31}{6.625 \times 10^{23}}(529.5\ K - 353\ k) = 3.32 \times 10^{-21} Joule = 0.02eV$$

6- Change in entropy at constant pressure

$$\Delta S = S_2 - S_1 = n\ C_P\ ln\left(\frac{T_2}{T_1}\right) = 16.57\ mole \times 37.395 \frac{J}{moleK} \times ln\left(\frac{529.5\ K}{353\ k}\right) = 251.24\ \frac{J}{K} = 60.1 \frac{Cal}{K}$$

7-Change of internal energy is determined from

$$dU = n\ C_V\ dT = 16.57\ mole \times 29.085 \frac{J}{moleK}\ (529.5\ K - 353\ k) = 85062.136\ J = 20349.79\ C = 20.349KC$$

8- Mechanical work for isobaric process is

$$W = P(V_2 - V_1) = 243 \times 10^5\ Pa\ (3 - 2) \times 10^{-3} m^3 = 24300\ J = 24.3\ KJ$$

The volume of the gas increases so the work is positive and don by the gas

9-the quantity of heat dQ can calculate by two method

a) $dQ = n\ C_P\ dT = 16.57\ mole \times 37.395 \frac{J}{moleK} \times (529.5\ K - 353\ k) = 109365.6\ J = 26164\ Ca = 26.164\ KC$

OR

b) $dQ = dU + dW = 20.349KC + \frac{24.3\ KC}{4.18} = 26.16\ KC$

Question (2) (12 marks)

2-a) Proofing the equation $PV^{\gamma} = constant$ in text book notes in page 82 .

Argon gas is mono-atomic gas where $\gamma = \frac{C_P}{C_V} = \frac{\frac{5}{2}R}{\frac{3}{2}R} = \frac{5}{3} = 1.667$

For suddenly changes in volume mean the processes is adiabatic where

$P_1V_1^{\gamma} = P_2V_2^{\gamma} = constant$ and $\Delta V = 0.4\ V_1$ this mean that $V_2 = 0.6\ V_1$

The ratio of pressures is $\frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^{\gamma} = \left(\frac{1}{0.6}\right)^{1.667} = 2.343 = 234.32\%$

2-b) Kinetic theory of gas explain the four laws of an ideal gas explained in text book page 104

2-c) Change of entropy and work during complete cycle of Carnot heat engine explained in text book page 127

$$\textbf{Efficiency } \eta = \frac{T_h - T_c}{T_c} = \frac{600 - 60}{600 + 273} = 0.618$$

And $\eta = \frac{W_{out}}{Q_{in}}$ so $W_{out} = \eta \times Q_{in} = 0.618 \times 5KC \times 4. = 12.9162\ KJ$

And $W_{out} = \text{acting force} \times \text{horizontal distance}$

$W_{out} = F_{acting} \times d$ so the required $F_{acting} = \frac{W_{out}}{d} = \frac{12.9162 \times 10^3\ J}{30\ m} = 430.54\ N$

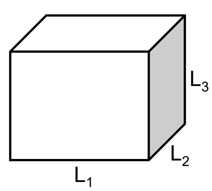
<u>Question (3) (20 marks)</u>

3-a) Derive the relation between the linear and volume coefficients of thermal expansion for isotropic solid materials? **(5 marks)**

Answer:

An isotropic solid is one in which the the coefficient of linear expansion is the same in all direction

Consider an isotropic solid of dimensions L_1, L_2 and L_3



Its volume, $V = L_1 L_2 L_3$

When the temperature is increased by ΔT, Its new volume will be,

$V + \Delta V = (L_1 + \alpha L_1 \Delta T)\ (L_2 + \alpha L_2 \Delta T)\ (L_3 + \alpha L_3 \Delta T)$

$V + \Delta V = L_1 (1 + \alpha\ \Delta T)\ \ L_2 (1 + \alpha\ \Delta T)\ \ L_3 (1 + \alpha\ \Delta T)$

$V + \Delta V = L_1 L_2 L_3 (1 + \alpha\ \Delta T)^3$

$V + \Delta V = V (1 + \alpha\ \Delta T)^3$

$V + \Delta V = V [1 + 3 (\alpha\ \Delta T) + 3 (\alpha\ \Delta T)^2 + (\alpha\ \Delta T)^3]$

as α is very small so, $(\alpha\ \Delta T)^2$ and $(\alpha\ \Delta T)^3$ are neglected

$V + \Delta V = V [1 + 3 (\alpha\ \Delta T)]$

$V + \Delta V = V + 3\ V\ \alpha\ \Delta T$

$\Delta V = 3\alpha\ V\ \Delta T$

Comparing with, $\Delta V = \beta\ V\ \Delta T$

So, $\beta = 3\alpha$

3-b) An eyeglasses frame is made of epoxy plastic. At room temperature (20 ^oC), the frame has circular lens holes 2.5 cm in radius. To what temperature must the frame be heated if lenses 2.513 cm in radius are to be inserted in it? The average coefficient of linear expansion of epoxy is $1.3\times10^{-4}\ {}^oC^{-1}$.

(5 marks)

Answer:

$T_i = 20\ {}^oC$

$\alpha = 1.3\times10^{-4}\ {}^oC^{-1}$.

$R = 2.5$ cm

$\Delta R = 2.513 - 2.5 = 0.013$ cm

$$\Delta R = \alpha\, R\, (T_f - T_i)$$

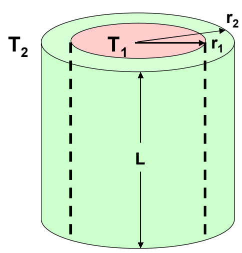
$$0.013 = 1.3\times10^{-4} \times 2.5 \times (T_f - 20)$$

$T_f = 60\ {}^oC$

3-c) Derive the equation of heat conduction through a pipe line of length (L) and inner and outer radii (r_1) and (r_2) if a hot liquid of temperature (T_1) passes inside the pipe while it surrounded by cold air at temperature (T_2)? **(5 marks)**

Answer:

- **Here, a pipe line of length (L) and inner and outer radii of (r_1) and(r_2) with $T_1 > T_2$**
- **Consider an element of length (L) radius (r) with thickness (dr). Then,**

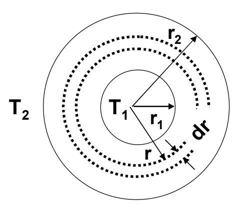


$$H = K\,A\,(-\frac{dT}{dr})$$

$$H = K\,(2\pi\, r\, L)\,(-\frac{dT}{dr})$$

$$\Rightarrow \quad \frac{dr}{r} = -\frac{K\,(2\pi L)}{H}\, dT$$

$$\Rightarrow \quad \int_{r_1}^{r_2} \frac{dr}{r} = -\frac{K\,(2\pi L)}{H} \int_{T_1}^{T_2} dT$$



$$\int_{r_1}^{r_2} \frac{dr}{r} = -\frac{K\,(2\pi L)}{H} \int_{T_1}^{T_2} dT$$

$$\Rightarrow \quad {}_{r_1}^{r_2}[\ln r] = -\frac{K\,(2\pi L)}{H}\, [T]_{T_1}^{T_2}$$

$$\Rightarrow \quad [\ln r_2 - \ln r_1] = -\frac{K\,(2\pi L)}{H}\, [T_2 - T_1]$$

$$\Rightarrow \quad \ln\frac{r_2}{r_1} = \frac{K\,(2\pi L)}{H}\, [T_1 - T_2]$$

$$\Rightarrow \quad \boxed{H = \frac{2\pi\, L\, K\,(T_1 - T_2)}{\ln\frac{r_2}{r_1}}}$$

3-d) A distant star appears blue with wavelength of 480 nm, **(5 marks)**

1) Estimate the surface temperature of that star?

Answer:

$$\lambda = \frac{0.2898}{T} \quad cm$$

$\lambda = 480x10^{-7}$ cm

So, $T = 0.2898 / 480x10^{-7}$

$= 6038$ K

2) Find the power of thermal radiation by that star to its surrounding space of a hypothetical temperature 50 ^{o}C? Assume a black body radiation, $\sigma = 5.67x10^{-8}$ $Wm^{-2}K^{-4}$.

Answer:

$T = 6038$ K $\quad$ $To = 50 + 273 = 323$ K

$\sigma = 5.67x10^{-8}$ $Wm^{-2}K^{-4}$

$e = 1$

$$P = e\,\sigma\,(T^4 - To^4)$$

$$= 1 \times 5.67x10^{-8}\,[(6038)^4 - (323)^4]$$

$$= 75.4x10^{6} \quad W\,m^{-2}$$

Question (4) (25 Marks)

4-a) If the irradiance of light is four time the incident irradiance when it pass through a Laser amplifier of 2 m active length, **i-** calculate β assuming no losses **ii-**If there were only a 8 % increase in irradiance , what would β be for the same value of Z? (4 mark)

Solution

i) Using the equation $I = I_o e^{\beta z}$

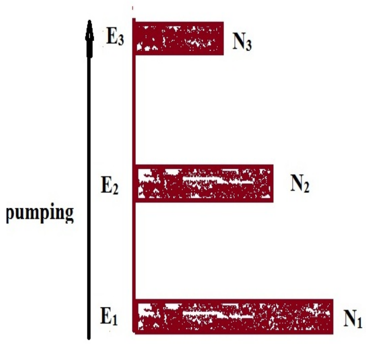
$I = 4 I_o$ $\quad$ $I/I_O = 4 = e^{\beta z}$ $\quad$ then $\ln 4 = \beta Z$ $\quad$ the $\beta = \ln 4 /2 = 0.69$ m^{-1}

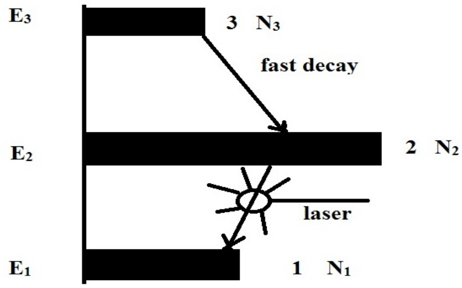
ii) $I = I_o + 0.08 I_o = I_o (1+0.08) = 1.08 I_o$

$I/I_o = 1.08 = e^{\beta z}$ $\quad$ then $\ln (1.08) = \beta Z$ $\quad$ then $\beta = \ln 1.08 /2 = 0.038$ m^{-1}

4-b) Explain in detail three level laser, why the two level laser is forbidden?

<u>Three level system (7 mark)</u>





a b

The distribution of atomic state population obeys Boltzmann's law (Fig. a).

By pumping with Xenon flash lamp, a large no. of atoms can be excited from $E_1 \rightarrow E_3$ and decay fastly to E_2, Fig b) .

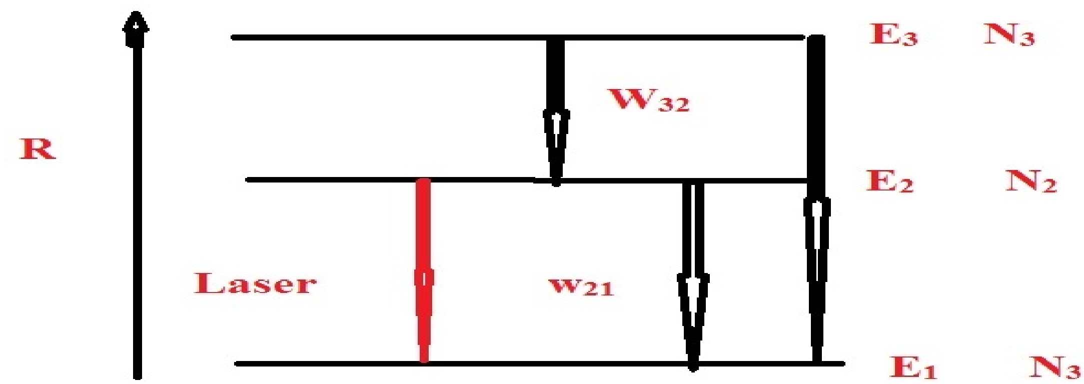


Fig. c

In fig. c

R the pumping rate

W_{21} the decay rate from level 2 to 1

W_{32} the decay rate from level 3 to 2

W_{31} the decay rate from level 3 to 1

The level 2 is called metastable level and the laser action is happened between level 2 and 1 . Assume that the total nuber of atom N is given by:

$N= N_1 + N_2 +N_3$

The transmission rate in three energy level is given y

$$\frac{dN_1}{dt} = -RN_1 + W_{21}N_2 + W_{31}N_3$$

$$\frac{dN_2}{dt} = -W_{21}N_2 + W_{32}N_3$$

$$\frac{dN_3}{dt} = RN_1 - W_{31}N_3 - W_{32}N_3$$

For stable state and for continuous pumping the number of atoms the number of atoms in all energy level are constant the rate equal zero

Then

$$\frac{dN_1}{dt} = -RN_1 + W_{21}N_2 + W_{31}N_3 = 0$$

$$\frac{dN_2}{dt} = -W_{21}N_2 + W_{32}N_3 = 0$$

$$\frac{dN_3}{dt} = RN_1 - W_{31}N_3 - W_{32}N_3 = 0$$

Then

$$RN_1 = W_{21}N_2 + W_{31}N_3$$

$$W_{21}N_2 = W_{32}N_3$$

$$RN_1 = (W_{31} + W_{32})N_3$$

Solving the last equation we get:

$$N_1 = \frac{W_{21}(W_{31} + W_{32})}{W_{21}(W_{31} + W_{32}) + (W_{32} + W_{21})R} N$$

Using the same way to calculate N_2 we get

$$N_2 = \frac{W_{32}R}{W_{21}(W_{31} + W_{32}) + (W_{32} + W_{21})R} N$$

$$\frac{N_2}{N_1} = \frac{W_{32}R}{W_{21}(W_{32} + W_{31})}$$

$$\frac{N_2}{N_1} = \frac{R}{W_{21}/W_{32}(W_{32} + W_{31})}$$

$$\frac{N_2}{N_1} = \frac{R}{W_{21}\left(1 + \frac{W_{31}}{W_{32}}\right)}$$

For more intense pumping and to achieve population inversion N_2 must be greater than N_1 , then

$$R > W_{21}\left(1 + \frac{W_{31}}{W_{32}}\right)$$

It is clear from the last equation to get population inversion the value of W_{21} is very small (the decay from level 1 to 2) , W_{32} is greater than W_{31} . This mean the decay from level 2 to level 1 must be slower and the decay from level 3 to level 2 must be very fast.

For two level system, the atoms are excited by optical pumping or collisions of atoms with the electrons. The number of atom in the excited state are increases. However, in the same, there is a fast decay from the excited to ground state as a result of the closer the two energy state. **Therefore, a population inversion in a two – level system (laser) can never be achieved**.

4-c) Write Einstein's equations for stimulated absorption spontaneous emission and stimulated emission. Also find the relation between Einstein coefficients (6 mark)

Answer :

If the number of atoms in the ground sate N_1 and in the excited state N_2

The transition rate in each of the case is given by.

Spontaneous emission transition rate = $N_2 A_{21}$ (1)

Stimulated absorption transition rate = $N_1 \rho_\upsilon B_{12}$ (2)

Stimulated emission transition rate = $N_2 \rho_\upsilon B_{21}$ (3)

Where A_{21} , B_{12} and B_{21} are Einstein coefficient for spontaneous emission , stimulated absorption and stimulated emission respectively. ρ_υ is the energy density $\rho_\upsilon = \frac{I_\upsilon}{4\pi c}$,

I_υ is the intensity of the beam.

$$\tau_{21} = \frac{1}{A_{21}}$$ is the transition life time

The above coefficient can be determined using the thermal equilibrium condition. In black body radiation, Einstein assume that the energy density of electromagnetic wave as a function of frequency and temperature is given by

4) ($$\rho_\upsilon = \frac{8\pi h \upsilon^3}{c^3}\left(\frac{1}{e^{h\upsilon/KT} - 1}\right)$$

At thermal equilibrium the number of atoms in the ground and excited state are constant.

Then

5) ($$N_2 A_{21} + N_2 B_{21} \rho_\upsilon = N_1 \rho_\upsilon B_{12}$$

And

(6) $$\rho_\upsilon = \frac{N_2 A_{21}}{N_1 B_{12} - N_2 B_{21}}$$

The last equation can be written as

(7) $$\rho_\upsilon = \frac{A_{21}/B_{21}}{N_1 B_{12}/N_2 B_{21} - 1}$$

But the ratio between N_1 and N_2 according to Maxwell distribution is given by.

)8 ($$\frac{N_1}{N_2} = e^{[(E_2 - E_1)/KT]}$$

Where $E_2 - E_1 = h\upsilon$

then

(9) $$\frac{N_1}{N_2} = e^{h\upsilon/KT}$$

Where K is Boltzmann constant , the

(10) $$\rho_\upsilon = \frac{A_{21}/B_{21}}{B_{12}/B_{21}\, e^{h\upsilon/KT} - 1}$$

The last equation take the form

(11) $$\rho_\upsilon = \frac{A_{21}}{B_{21}}\left(\frac{1}{\frac{B_{12}}{B_{21}}\left(e^{h\upsilon/KT} - 1\right)}\right)$$

Comparing equation 4 and equation 11 we get

(3-12) $$B_{12} = B_{21}$$

و(3-13) $$A_{21} = B_{21} \frac{8\pi h \upsilon^3}{c^3}$$

4-d) a concave spherical mirror forms an inverted image 4 times larger than the object. Assuming the distance between object and image is 60 cm, find the focal length of the mirror. Suppose the mirror is convex. The distance between the image and the object is 60 cm, but the image is 0.5 the size of object. Determine the focal length of the mirror. (5 – mark)

Solution

For concave $m = s'/s = 4$ the $S' = 4S$

$S' - S = 60$ them $4S - S = 60$ then $S = 20$ cm & $S' = 80$

But $1/S = 1/S' = 1/f$ then $f = 16$ cm

For convex $S + S' = 60$ & $S'/S = 0.5$ $S' = 0.5 S$

Then S 40 cm Applying the general mirror law we get $f = -40$ cm

4-e) A glass optical fiber (n= 1.50) is submerged in water (n = 1.33). What is the critical angle for light to stay inside the fiber? (3 mark)

$n_f \sin \theta_{cf} = n_w \sin 90$

$1.5 \sin \theta_{cf} = 1.33$ $\sin \theta_{cf} = 1.33/1.5$ then $\theta_{cf} = 62.3^o$

5-a) (**5 marks**)

i- Parallel conductors carrying current in the same direction **attract** each other.

ii- The torque acting on current loop has its maximum value when the field is **parallel to** the plane of the loop.

iii- A **mass spectrometer** separates ions according to their mass-to-charge ratio.

iv- Ampere's law states that **the line integral of B.ds around any closed path equals μ_0 $i_{enclosed}$**

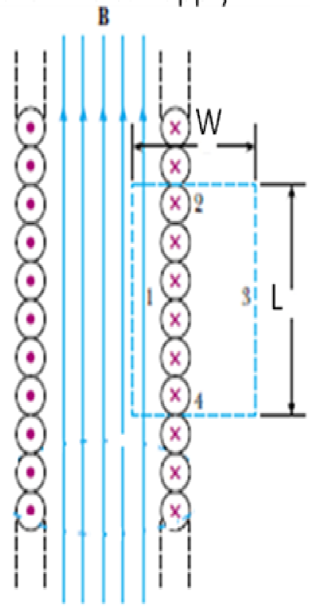
Where $i_{enclosed}$ is the total steady current passing through any surface bounded by the closed path

$$\oint B.ds = \oint B ds \cos\theta = \mu_0 i_{enclosed}$$

5-b) (**5 Marks**)

Magnetic Field inside a Solenoid carrying current i

Consider the rectangular path of length L and width w shown in Figure. We can apply Ampere's law to this path



$$\because \oint B ds \cos\theta = \mu_0 i_{enclosed}$$ $\boxed{\text{ampere's law}}$

L.H.S

$$\oint B ds \cos\theta = \int_{path1} B ds \cos\overset{\theta=0}{\theta} + \underbrace{\int_{path2} B ds \cos\theta}_{0} + \underbrace{\int_{path3} B ds \cos\theta}_{0} + \underbrace{\int_{path4} B ds \cos\theta}_{0}$$

$$= \int_{path1} B ds = B\int_0^L ds = BL$$

R.H.S $\quad \therefore \mu_0 i_{enclosed} = \mu_0 N i$

The total current through the rectangular path equals the current through each turn multiplied by the number of turns

Left hand side = right hand side

$$BL = \mu_0 N i$$

$$\therefore B = \frac{\mu_0 N i}{L} = \mu_0 n i$$

Where n= N/L is the number of turns per unit length

5-c) (**5 marks**)

Given:

N=250 turns

A=200x10^{-4} =0.02 m^2

R=3Ω

at t=0 , Φ_B=0 wb because B=0 T at this time ,

at t= 1.2 s , Φ_B=BA=2T x 0.02 m^2=0.04 wb

The magnitude of the induced electromotive force (ε) is given by:

$$|\varepsilon| = N\frac{\Delta\phi_B}{\Delta t} = 250\frac{0.04wb - 0wb}{1.2s - 0s} = 8.33V$$

Apply ohm's law we get the induced current in the coil

5-d) (**5 marks**)

Consider the long solenoid as circuit 1 . Acurrent i_1
In it set up afield at its center

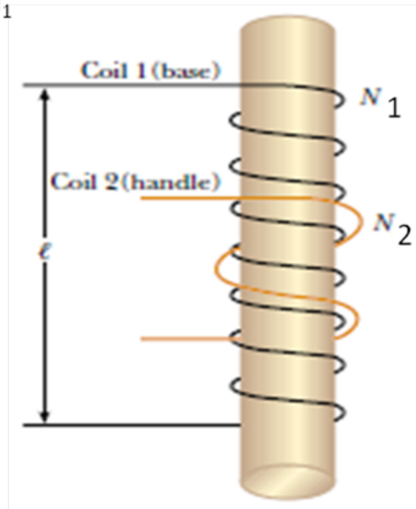
$$B_1 = \frac{\mu_0 N_1 i_1}{l}$$

The flux through the central section is

$$\phi_{12} = BA = \frac{\mu_0 N_1 i_1}{l} A$$

Since this flux links with coil 2

$$\therefore M_{12} = N_2 \frac{\phi_{12}}{i_1} == \frac{\mu_0 N_1 N_2}{l} A$$



Given: (L=1 m, A=$10x10^{-4}$ m^2, N_1=1000 turns,N_2=20 turns)

$$i- \therefore M = \frac{4\pi x 10^{-7} x 1000 x 20}{1} 10^{-3} = 25.1 x 10^{-6}\, henery$$

$$ii- \therefore \varepsilon_2 = -M \frac{di_1}{dt} = -25.1 x 10^{-6} x 10 \quad V$$